Physics 3A for Electrical Engineering

- Home assignments: Mandatory to submit 10 out of 10-13 that will be published.
- The final grade will be the number of submitted home assignments (up to 10) + 0.9* grade of the final exam.
- Information about the structure of the final exam will be published toward the end of the course.
- Reception hours: Wednesday 14:00-15:00 zoom link published on the course website.
- My email for questions: bel@bgu.ac.il

Basic Elements of Quantum Theory

- The photoelectric effect
- Waves
- de Broglie wavelength
- Schrodinger equation
- Probability and its relation to the wavefunction
- Expectation values of operators
- Heisenberg uncertainty principle

Recommended literature:

- *Basic Quantum Mechanics*, J. M. Cassels, Chapters 1-3, 7.
- *Introduction to Modern Physics*, J. D. McGervey, Chapters 3-5, 10.
- *Solid State Physics*, N. W. Ashcroft and N. D. Mermin, Chapters 1-5, 8-9.
- Lecture notes by Emir Erez, look for the link on the course web page.

Waves

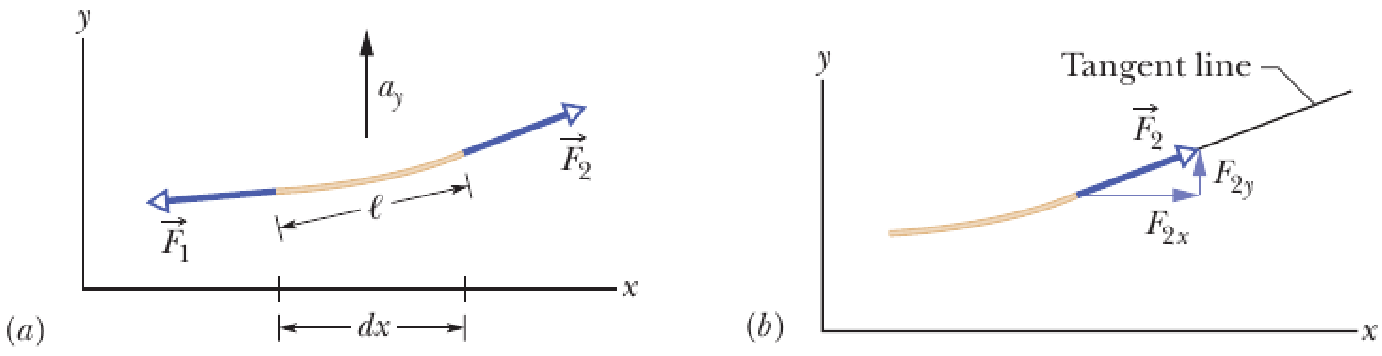
- Wave is the propagation of dynamic disturbance of a physical quantity.
- A periodic wave is the propagation of a periodic fluctuation in space and time.
- Longitudinal wave – when the oscillation is in the direction of propagation (e.g., sound waves).
- Transverse wave – when the oscillation is perpendicular to the direction of propagation (e.g., surface ocean waves).
- The classical 1D wave equation is

$$\frac{\partial^2 \psi(x,t)}{\partial t^2} = c^2 \frac{\partial^2 \psi(x,t)}{\partial x^2}$$

c – is the propagation speed of the wave

1D-Elastic wave equation

- Consider a string which at rest is along the x-axis.
- When a fluctuation along the y-axis is initiated the string tension applies forces to each segment of the string.
- Under the assumption of small fluctuations the dynamics is along the y-axis only.
- Using Newton's law we can derive the equation of motion for each segment.
- The linear density is $\rho = M/L$, where M is the mass of the entire string and L is its length at rest (parallel to the x-axis).
- The forces at the ends of the segment are, in general, not parallel because of the angle of the string.
- The vertical only motion implies no acceleration in the x-direction.



1D-Elastic wave equation

- Newton's 2nd law for the two components reads:

$$T(x,t)cos\big(\theta(x,t)\big) = T(x+\Delta x,t)cos\big(\theta(x+\Delta x,t)\big) = constant = \tau$$

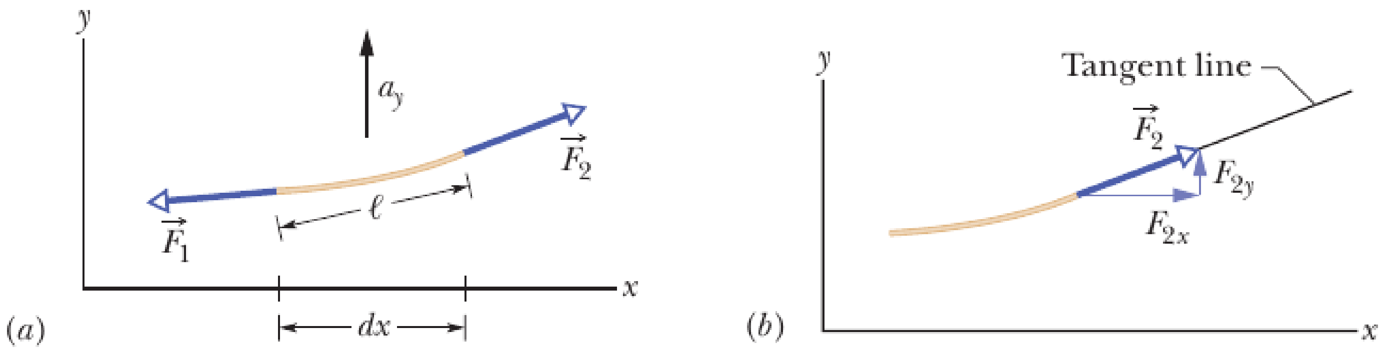
The above equation implies a constant horizontal tension in the string.

$$dm\frac{\partial^2 y}{\partial t^2} = T(x+\Delta x,t)sin\big(\theta(x+\Delta x,t)\big) - T(x,t)sin\big(\theta(x,t)\big)$$

$$= T(x+\Delta x,t)cos\big(\theta(x+\Delta x,t)\big)tan\big(\theta(x+\Delta x,t)\big) - T(x,t)cos\big(\theta(x,t)\big)tan\big(\theta(x,t)\big)$$

$$= \tau\Big(tan\big(\theta(x+\Delta x,t)\big) - tan\big(\theta(x,t)\big)\Big)$$

- Note that $\tan(\theta(x,t)) = \frac{\partial y}{\partial x}$
- $dm\frac{\partial^2 y}{\partial t^2} = \tau\left(\frac{\partial y}{\partial x}(x+\Delta x,t) - \frac{\partial y}{\partial x}(x,t)\right)$



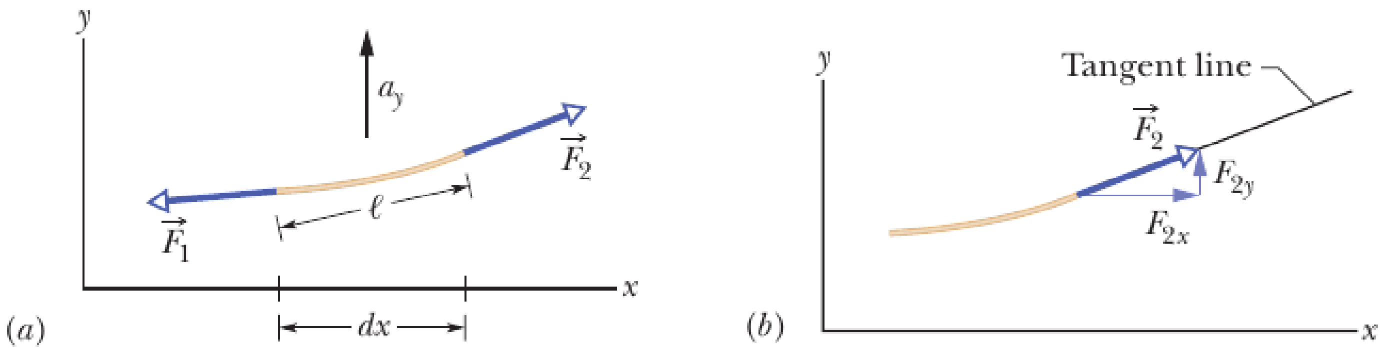
1D-Elastic wave equation

- Restricting ourselves to small angles, i.e., small vertical fluctuations, we may write

$$\rho \Delta x \frac{\partial^2 y}{\partial t^2} = \tau \left(\frac{\partial y}{\partial x}(x + \Delta x, t) - \frac{\partial y}{\partial x}(x, t) \right)$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{\tau}{\rho} \frac{\partial^2 y}{\partial x^2}$$

Note that τ has the units of force $[mass \times length/time^2]$, the density has units of $[mass/length]$ so $\frac{\tau}{\rho}$ has the units of $\left[\left(\frac{length}{time}\right)^2\right] = [velocity^2]$.



Waves

- Solution by separation of variables

$$\psi(x,t) = X(x)T(t)$$

$$X(x)\frac{\partial^2 T(t)}{\partial t^2} = c^2 T(t)\frac{\partial^2 X(x)}{\partial x^2}$$

$$\frac{1}{c^2 T(t)}\frac{\partial^2 T(t)}{\partial t^2} = \frac{1}{X(x)}\frac{\partial^2 X(x)}{\partial x^2} = const = -k^2$$

Note that k is independent of x and t, and it has units of 1/length

$$X(x) = A_1 e^{ikx} + A_2 e^{-ikx} \qquad T(t) = B_1 e^{ikct} + B_2 e^{-ikct}$$

$$\psi(x,t) = A_+ \cos(kx - \omega t + \phi_+) + A_- \cos(kx + \omega t + \phi_-)$$

$$\omega = ck$$

The unknown coefficients are determined by the boundary and initial conditions.

The solution is a superposition of waves traveling in the positive and negative directions.

Waves

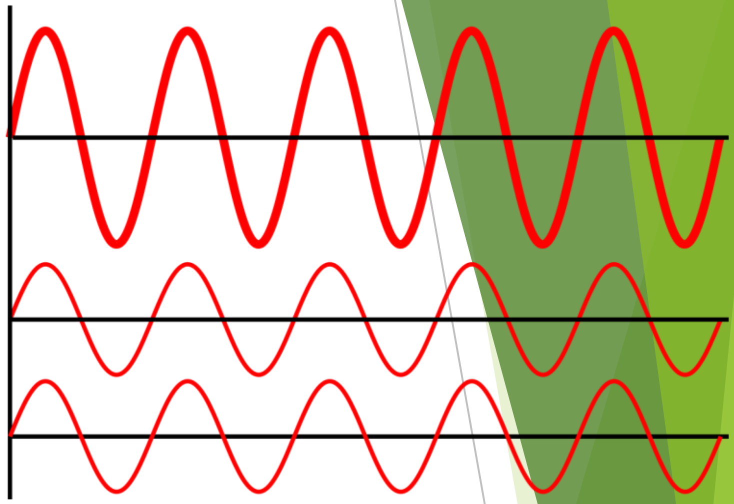
- $\psi(x,t) = A_+ \cos(kx - \omega t + \phi_+) + A_- \cos(kx + \omega t + \phi_-)$
- $\omega = ck$
- The solution is a superposition of waves traveling in the positive and negative directions. Due to the linear nature of the equation the sum of any number of solutions is also a solution (superposition).
- The solution is invariant to time translations by $\tau = \frac{2\pi n}{\omega} = nT$ where $T = \frac{2\pi}{\omega} = 1/\nu$ is the period of the wave.
- The solution is invariant to translations by $l = \frac{2\pi n}{k} = n\lambda$ where $\lambda = 2\pi/k$ is the wavelength.
- k is the wavenumber

$kx \pm \omega t = k(x \pm v_p t)$ so it's clear that the phase velocity is $v_p = \frac{\omega}{k} = c$

Waves

- At a given point in space, two waves may interfere

Constructive (in-phase)



The simplest example for a constructive interference at all points is

$$y(x,t) = A sin(kx - \omega t) + A sin(kx - \omega t + \phi)$$

Using

$$\sin(\alpha) + \sin(\beta) = 2\sin\left(\frac{1}{2}(\alpha+\beta)\right)\cos\left(\frac{1}{2}(\alpha-\beta)\right)$$

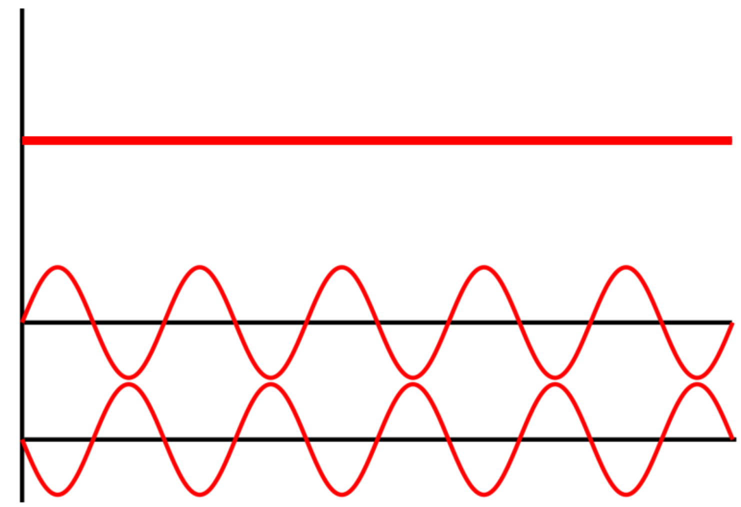
$$y(x,t) = 2A cos\left(\frac{1}{2}\phi\right) sin\left(kx - \omega t + \frac{1}{2}\phi\right) = 2A sin(kx - \omega t)$$

$$\phi = 2\pi n$$

Waves

- At a given point in space, two waves may interfere

Destructive (anti-phase)



The simplest example for a destructive interference at all points is

$$y(x,t) = Asin(kx - \omega t) + Asin(kx - \omega t + \phi)$$

Using

$$\sin(\alpha) + \sin(\beta) = 2\sin\left(\frac{1}{2}(\alpha+\beta)\right)\cos\left(\frac{1}{2}(\alpha-\beta)\right)$$

$$y(x,t) = 2Acos\left(\frac{1}{2}\phi\right) sin\left(kx - \omega t + \frac{1}{2}\phi\right) = 0$$

$$\phi = (2n+1)\pi$$

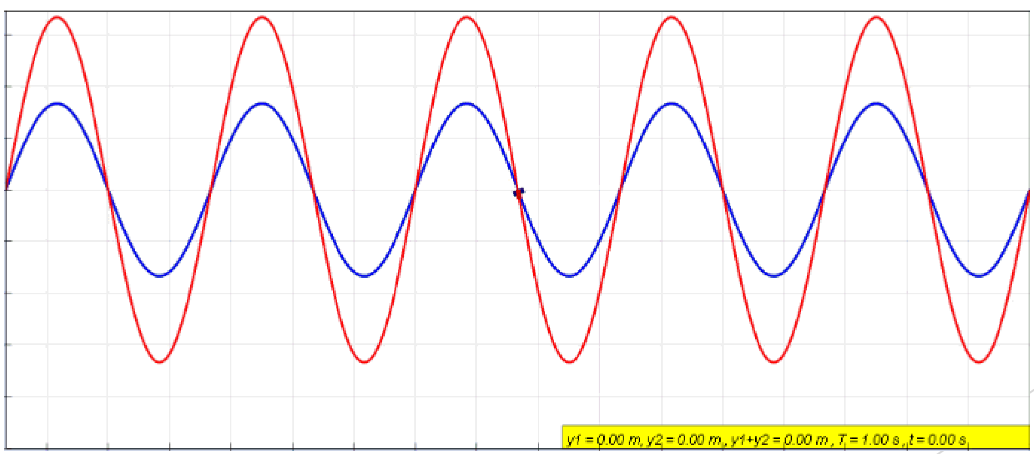
Waves

- Two harmonic waves with the same amplitude, traveling in opposite directions, yield standing waves

$$y(x,t) = A\sin\left(k\left(x - \frac{\omega}{k}t\right)\right) + A\sin\left(k\left(x + \frac{\omega}{k}t\right)\right)$$

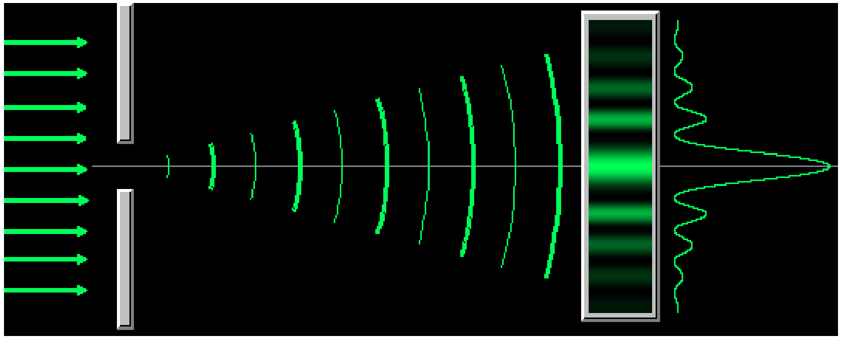
$$\sin(\alpha) + \sin(\beta) = 2\sin\left(\frac{1}{2}(\alpha+\beta)\right)\cos\left(\frac{1}{2}(\alpha-\beta)\right)$$

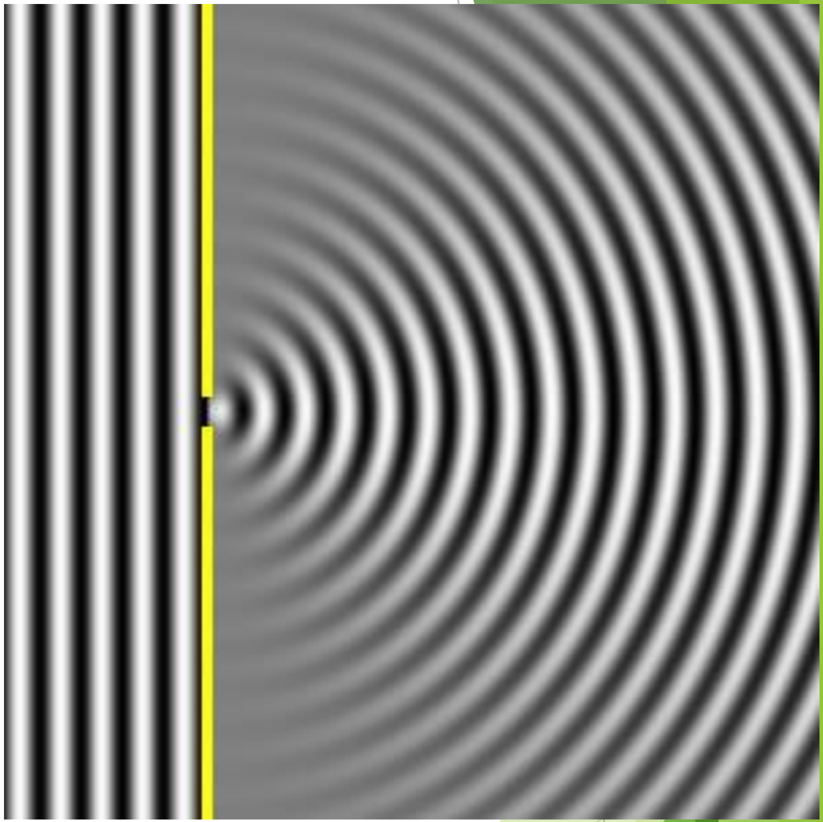
$$y(x,t) = 2A\cos(\omega t)\sin(kx)$$



By Lookang many thanks to author of original simulation = Wolfgang Christian and Francisco Esquembre author of Easy Java Simulation = Francisco Esquembre - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=39309437

Light passage through a single slit

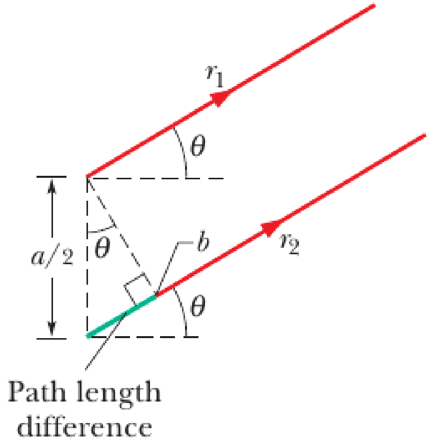




Lookangmany thanks to Fu-Kwun Hwang and author of Easy Java Simulation = Francisco Esquembre / CC BY-SA

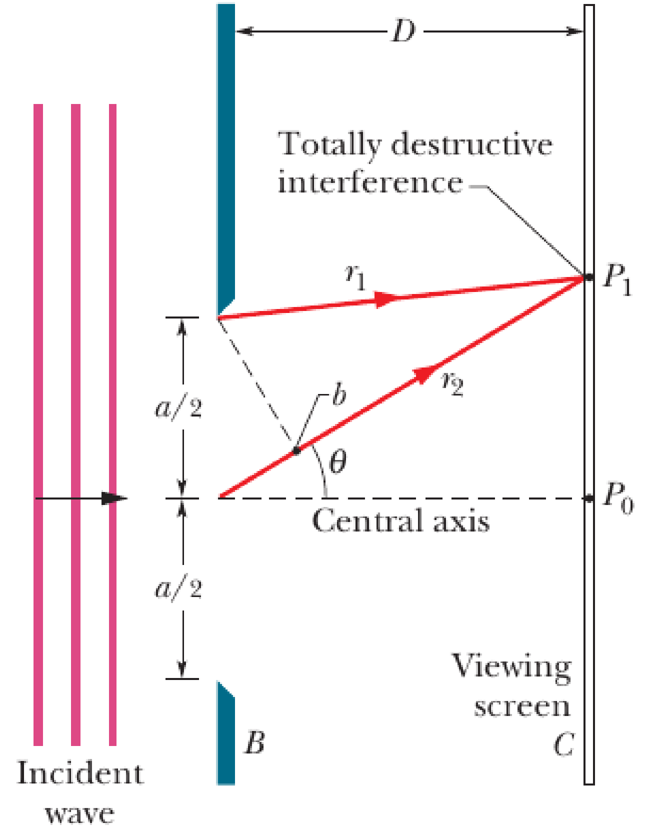
Huygens' principle – every point on a wave front is considered to be a secondary source of spherical wavelets.

Light passage through a single slit



This path length difference shifts one wave from the other, which determines the interference.

This pair of rays cancel each other at P_1. So do all such pairings.

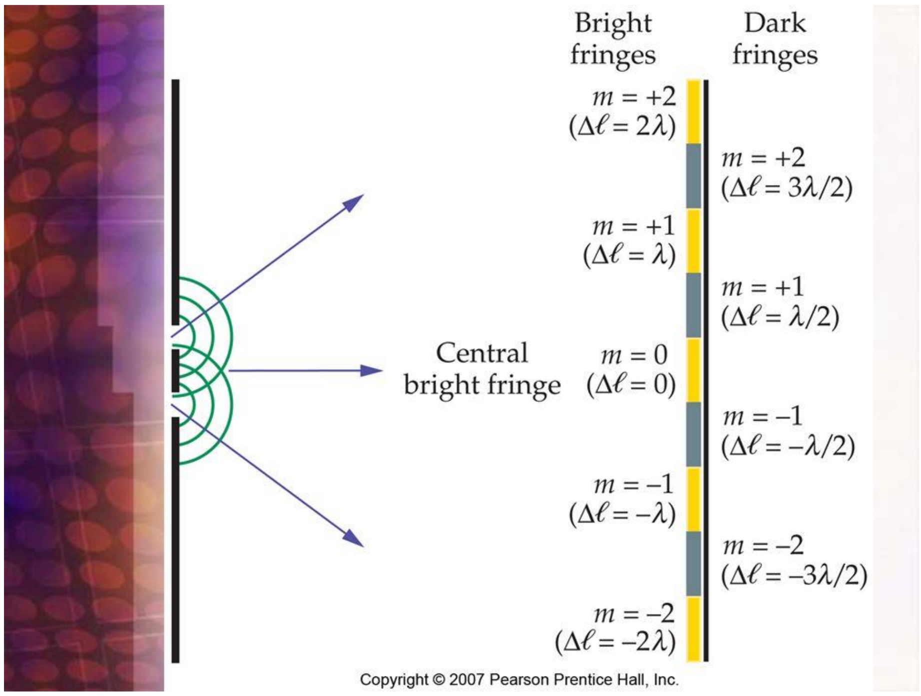


For $D >> a$, we can approximate rays r_1 and r_2 as being parallel, at angle θ to the central axis. The path length difference is then $\frac{a}{2}\sin(\theta)$. The condition for destructive interference is

$\frac{a}{2}\sin(\theta) = \frac{\lambda}{2}$. By repeating a similar analysis we find that the minima are found by

$$\sin(\theta) = \frac{m\lambda}{a}$$

Double slit diffraction



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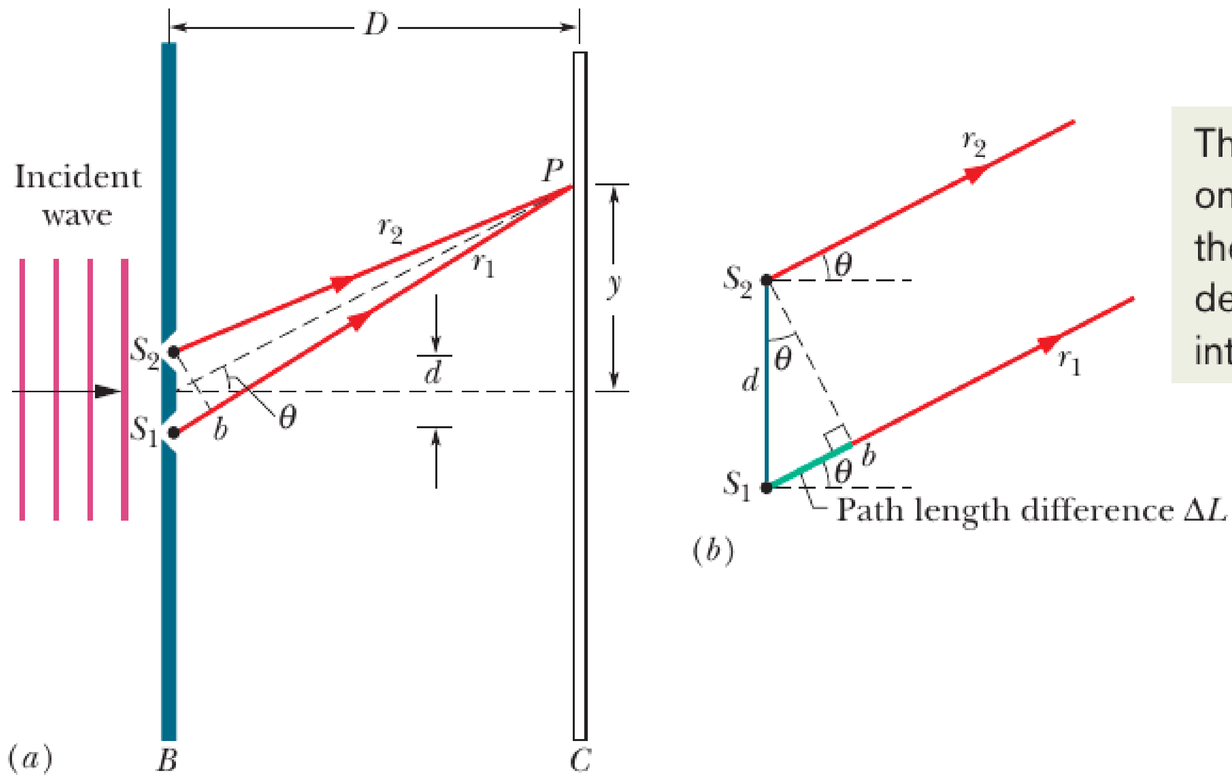
Double slit diffraction



The ΔL shifts one wave from the other, which determines the interference.

Qualitative explanation

- Let's assume that the electric field produced by waves originating at different points is in the same direction.
- $\vec{E} = E\,\vec{e} = \vec{E}_1 + \vec{E}_2 = E_1\,\vec{e}_1 + E_2\vec{e}_2$
- Assuming that both fields are in the same direction the intensity is proportional to
- $I \propto |\mathrm{E}|^2 = (E_1 + E_2)(E_1^* + E_2^*) = |E_1|^2 + |E_2|^2 + E_1^* E_2 + E_1 E_2^*$
- Far from the slit, each wave may be considered as a plane wave $E_1 = A_1 e^{i\phi_1(x)}; E_2 = A_2 e^{i\phi_2(x)}$ and for equal amplitudes

$$I \propto 2A_1^2\left(1 + \cos\left(\phi_1(x) - \phi_2(x)\right)\right) = 2A_1^2\left(1 + \cos\left(\Delta\phi(x)\right)\right)$$

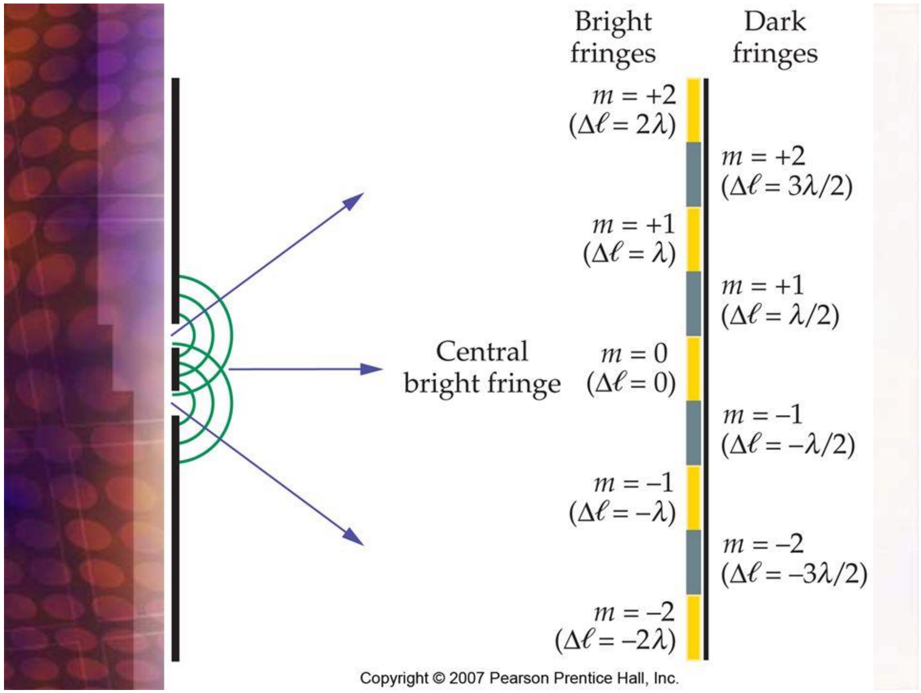
$$= 4A_1^2 \cos^2\left(\frac{\Delta\phi(x)}{2}\right)$$

Qualitative explanation

$$I \propto 4A_1^2 \cos^2\left(\frac{\Delta\phi(x)}{2}\right)$$

The intensity is maximal when the phase difference is equal to $2\pi n$

(for any integer n)



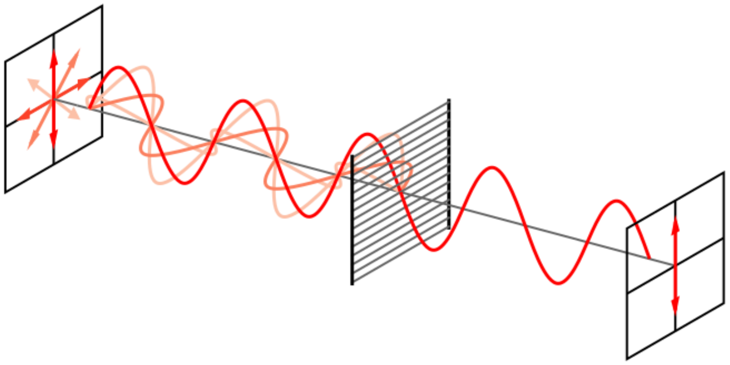
What happens when photons are passing one at a time?

- A particle is expected to go through a specific point in the slit. How can it interfere with itself?
- Under these conditions, each photon hits another location and the average intensity (of many single photons) results in the same diffraction image as many simultaneous photons.
- The same behavior was observed for electrons and is not limited to photons.

Polarization

- The electric field due to a linearly polarized light (in the x-y plane) may be written as

$$\vec{E}(z,t) = E_0 \vec{e}_p \cos(kz - \omega t)$$



- The unit vector of the polarization direction may be written as

$$\vec{e}_p = \cos(\phi)\ \hat{x} + \sin(\phi)\ \hat{y} = \cos(\phi)|x\rangle + \sin(\phi)\,|y\rangle$$

Polarization

- When the light is filtered by a polarizer, which only allows the field in the *x* direction to pass through, the intensity becomes $I \propto |E|^2 \propto E_0^2 \cos^2(\phi)$
- When photons hit the polarizer, one at a time, some photons may pass and others won't and after a large number of photons, *N*, hit the polarizer the average number of passing photons is proportional to $Ncos^2(\phi)$