Dimensionless analysis and significance of time scales

for self-watch

Lecture 3:

https://www.youtube.com/watch?v=eZmzmQW-fAA&list=PLbN57C5Zdl6j_qJA-pARJnKsmROzPnO9V&index=3&t=24s

Main points to pay attention to:

- While in linear systems there is only one time scale (exponential decay or growth), in nonlinear systems the dynamics can be dominated by several time scales

- Often these fast/slow time scales can be revealed by dimensionless analysis

- Fast time scales are often being neglected in physics, by the so-called adiabatic approximation, which yields a near-equilibrium behavior

- However, in reality, fast time scales may have dramatic implications

$$
\begin{aligned}
b\dot{\phi} &= -mg\sin\phi + mr\omega^2 \sin\phi\cos\phi \\
&= mg\sin\phi\left(\frac{r\omega^2}{g}\cos\phi - 1\right).
\end{aligned}
$$

$$\gamma = \frac{r\omega^2}{g}$$

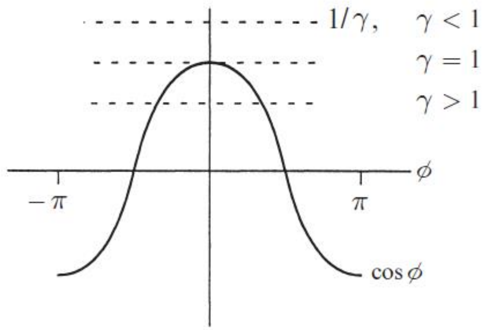
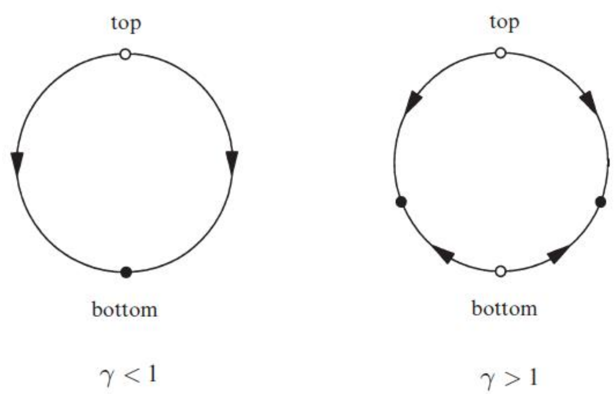
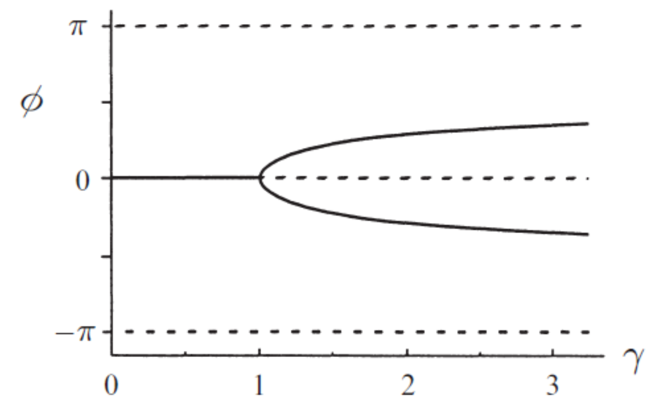


Figure 3.5.4

As $\gamma \to \infty$, these intersections approach $\pm\pi/2$. Figure 3.5.5 plots the fixed points on the hoop for the cases $\gamma < 1$ and $\gamma > 1$.





$$\tau = \frac{t}{T}$$

$$\frac{mr}{T^2}\frac{d^2\phi}{d\tau^2} = -\frac{b}{T}\frac{d\phi}{d\tau} - mg\sin\phi + mr\omega^2\sin\phi\cos\phi.$$

$$\left(\frac{r}{gT^2}\right)\frac{d^2\phi}{d\tau^2} = -\left(\frac{b}{mgT}\right)\frac{d\phi}{d\tau} - \sin\phi + \left(\frac{r\omega^2}{g}\right)\sin\phi\cos\phi.$$

$$\frac{b}{mgT} \approx O(1), \text{ and } \frac{r}{gT^2} << 1.$$

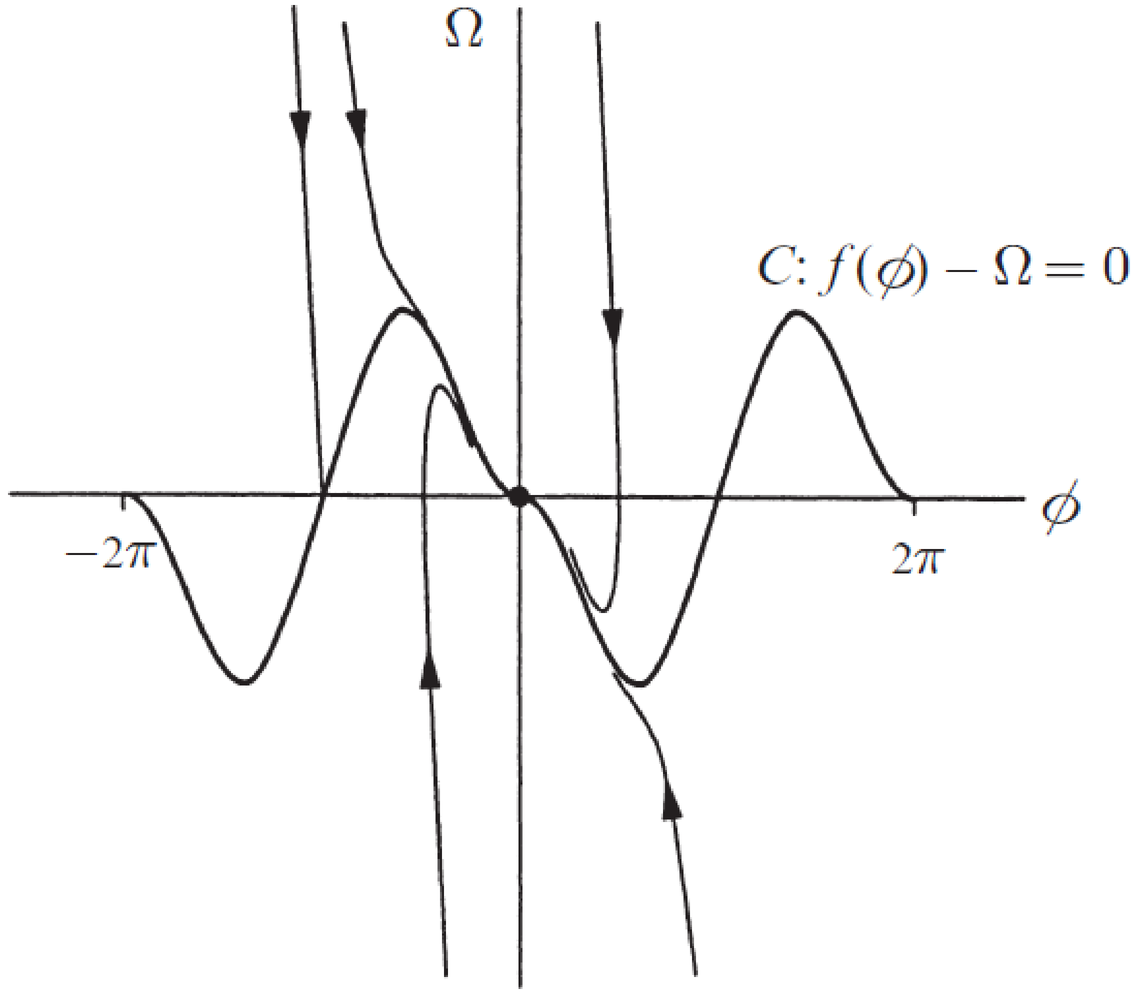
$$T = \frac{b}{mg}.$$

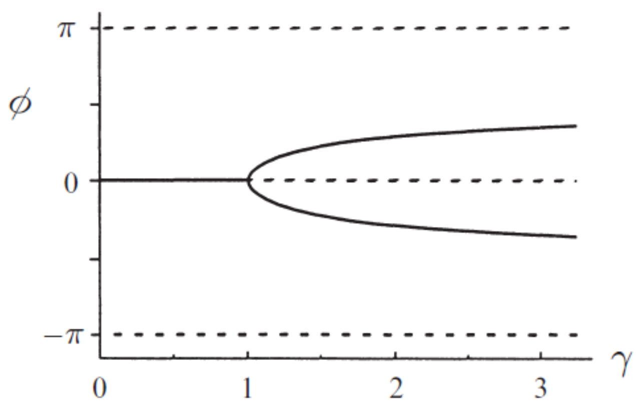
$$\frac{r}{g}\left(\frac{mg}{b}\right)^2 << 1,$$

$$b^2 >> m^2 gr.$$

$$\phi' = \Omega$$

$$\Omega' = \frac{1}{\varepsilon}(f(\phi) - \Omega).$$





Multiplicity of parameters and rescaling for self-watch

Lecture 4:

https://www.youtube.com/watch?v=P_YCvTabMO4&list=PLbN57C5Zdl6j_qJA-pARJnKsmROzPnO9V&index=4

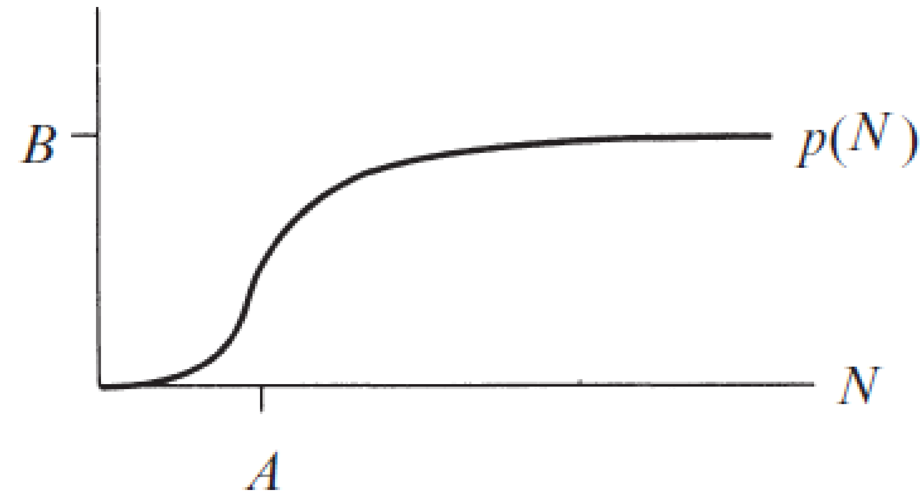
Main points to pay attention to:

- A multiplicity of parameters doesn’t necessarily imply complex dynamics

- The dynamics often can be revealed by looking at a single parameter, although it also can be a dimensionless parameter (a parameter that is a combination of the original ones)

- Other parameters act to quantitatively rescale the solution but they do not affect the dynamics

$$\dot{N} = RN\left(1 - \frac{N}{K}\right) - \frac{BN^2}{A^2 + N^2}.$$



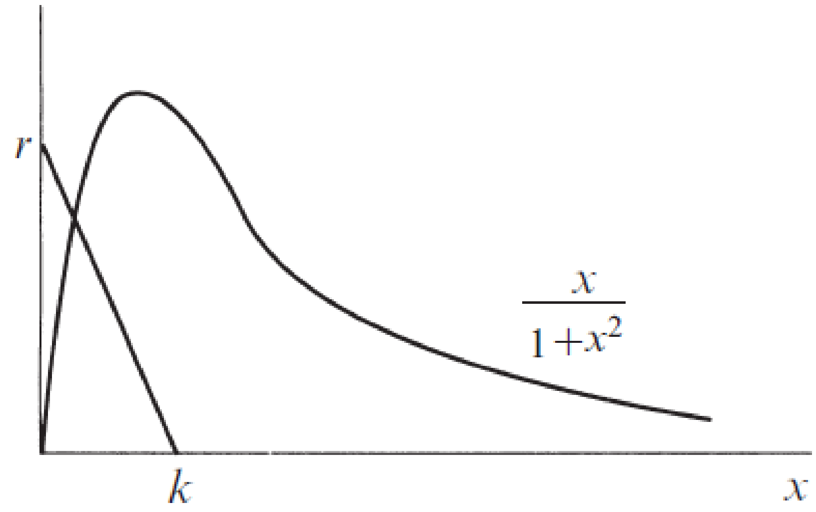
$$x = N/A,$$

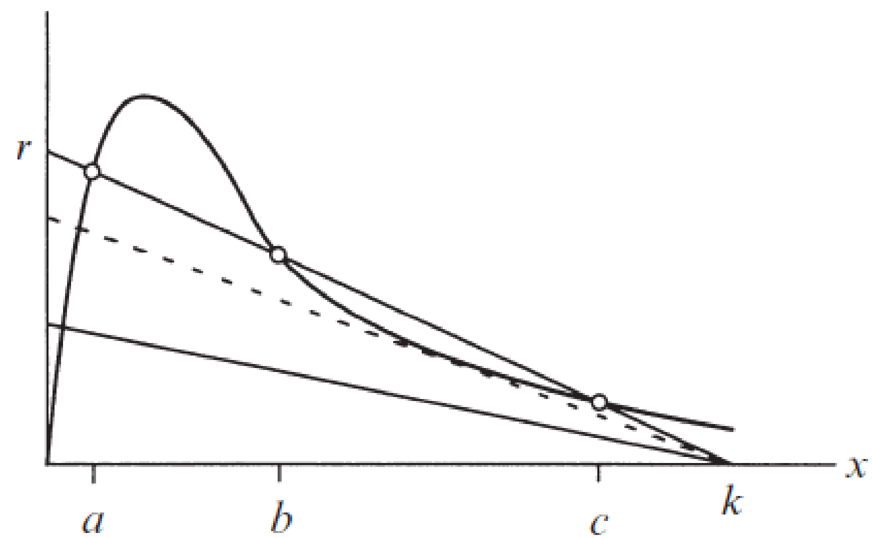
which yields

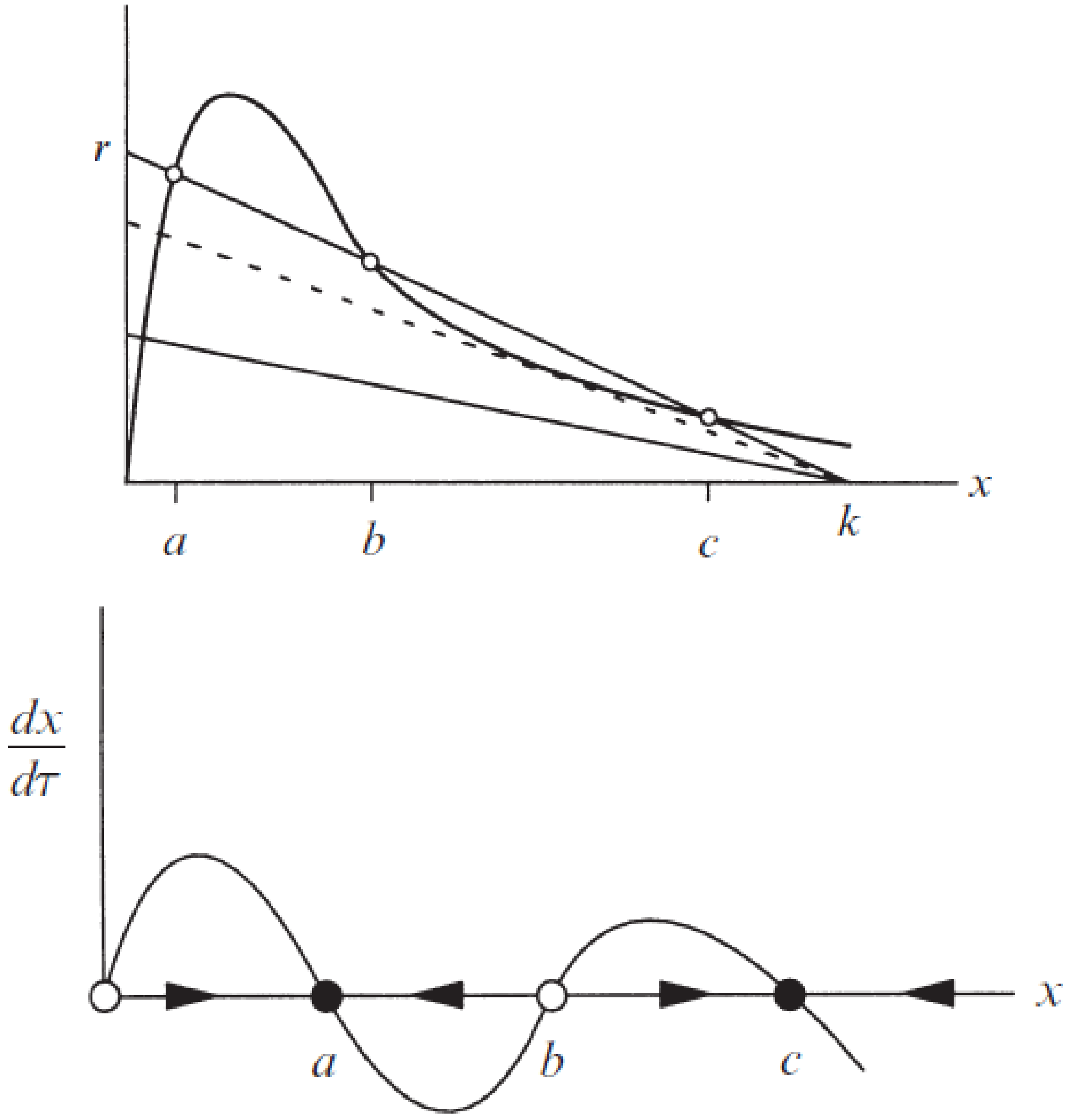
$$\frac{A}{B}\frac{dx}{dt} = \frac{R}{B}Ax\left(1 - \frac{Ax}{K}\right) - \frac{x^2}{1+x^2}. \qquad \tau = \frac{Bt}{A}, \qquad r = \frac{RA}{B}, \qquad k = \frac{K}{A}.$$

$$\frac{dx}{d\tau} = rx\left(1 - \frac{x}{k}\right) - \frac{x^2}{1+x^2},$$

$$r\left(1-\frac{x}{k}\right)=\frac{x}{1+x^2}$$

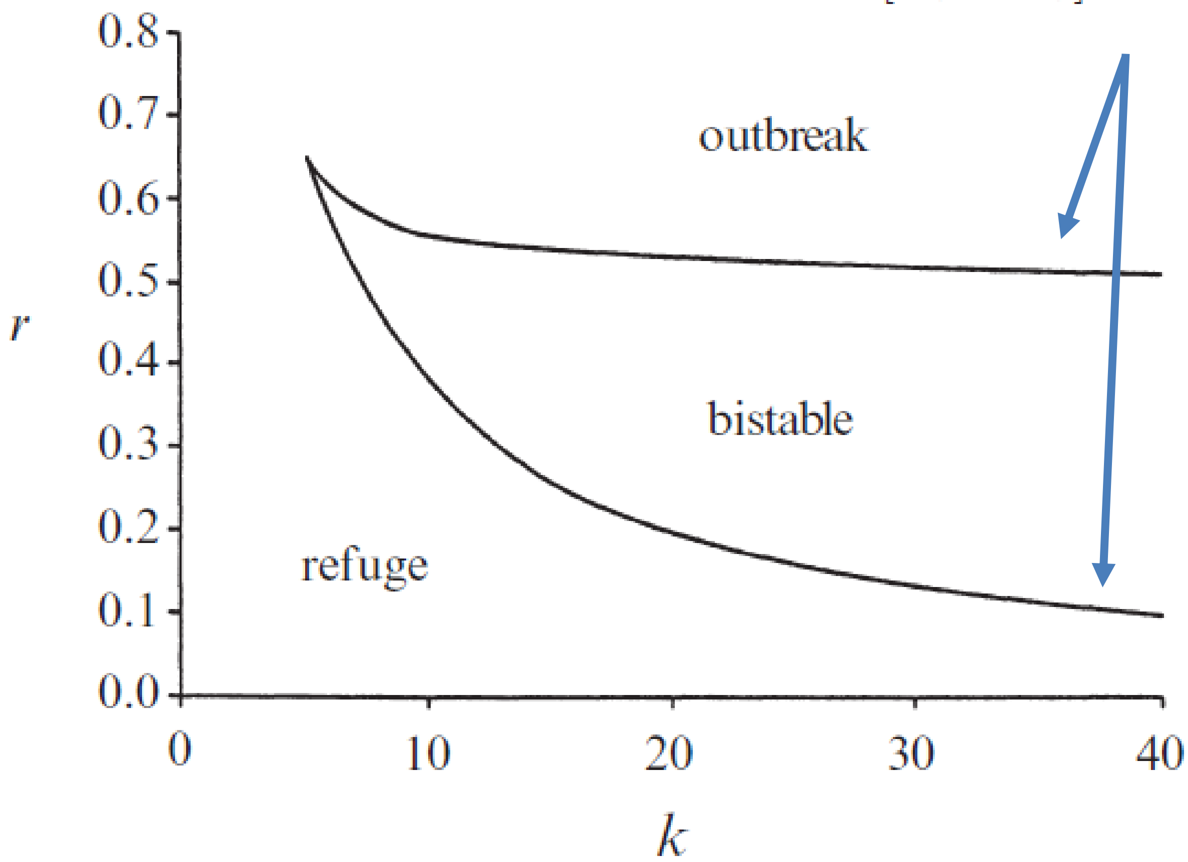





r
x
a
b
c
k
dx/dτ
x
a
b
c

$$\frac{d}{dx}\left[r\left(1-\frac{x}{k}\right)\right]=\frac{d}{dx}\left[\frac{x}{1+x^2}\right].$$

$$r=\frac{2x^3}{(1+x^2)^2}.$$



$$k=\frac{2x^3}{x^2-1}.$$

$$\frac{d}{dx}\left[r\left(1-\frac{x}{k}\right)\right]=\frac{d}{dx}\left[\frac{x}{1+x^2}\right].$$