

Suggested home exercises – lecture 5

6.5.1 Consider the system $\ddot{x} = x^3 - x$.

- Find all the equilibrium points and classify them.
- Find a conserved quantity.
- Sketch the phase portrait.

6.5.15 (Frictionless bead) Consider the undamped case $b = 0$.

- Show that the equation can be nondimensionalized to $\phi'' = \sin \phi (\cos \phi - \gamma^{-1})$, where $\gamma = r\omega^2/g$ as before, and prime denotes differentiation with respect to dimensionless time $\tau = \omega t$.
- Draw all the qualitatively different phase portraits as γ varies.
- What do the phase portraits imply about the physical motion of the bead?

6.6.1 $\dot{x} = y(1 - x^2)$, $\dot{y} = 1 - y^2$

Show that each of the following systems is reversible, and sketch the phase portrait.

6.7.1 (Damped pendulum) Find and classify the fixed points of $\ddot{\theta} + b\dot{\theta} + \sin\theta = 0$ for all $b > 0$, and plot the phase portraits for the qualitatively different cases.

7.1.3 $\dot{r} = r(1 - r^2)(4 - r^2)$, $\dot{\theta} = 2 - r^2$

Sketch the phase portrait for each of the following systems. (As usual, r, θ denote polar coordinates.)