Bifurcations in 2D

for self-watch

Lecture 12:

https://www.youtube.com/watch?v=oqKAVqe71vw&list=PLbN57C5Zdl6j_qJA-pARJnKsmROzPnO9V&index=12


in 2-D

Main points to pay attention to:

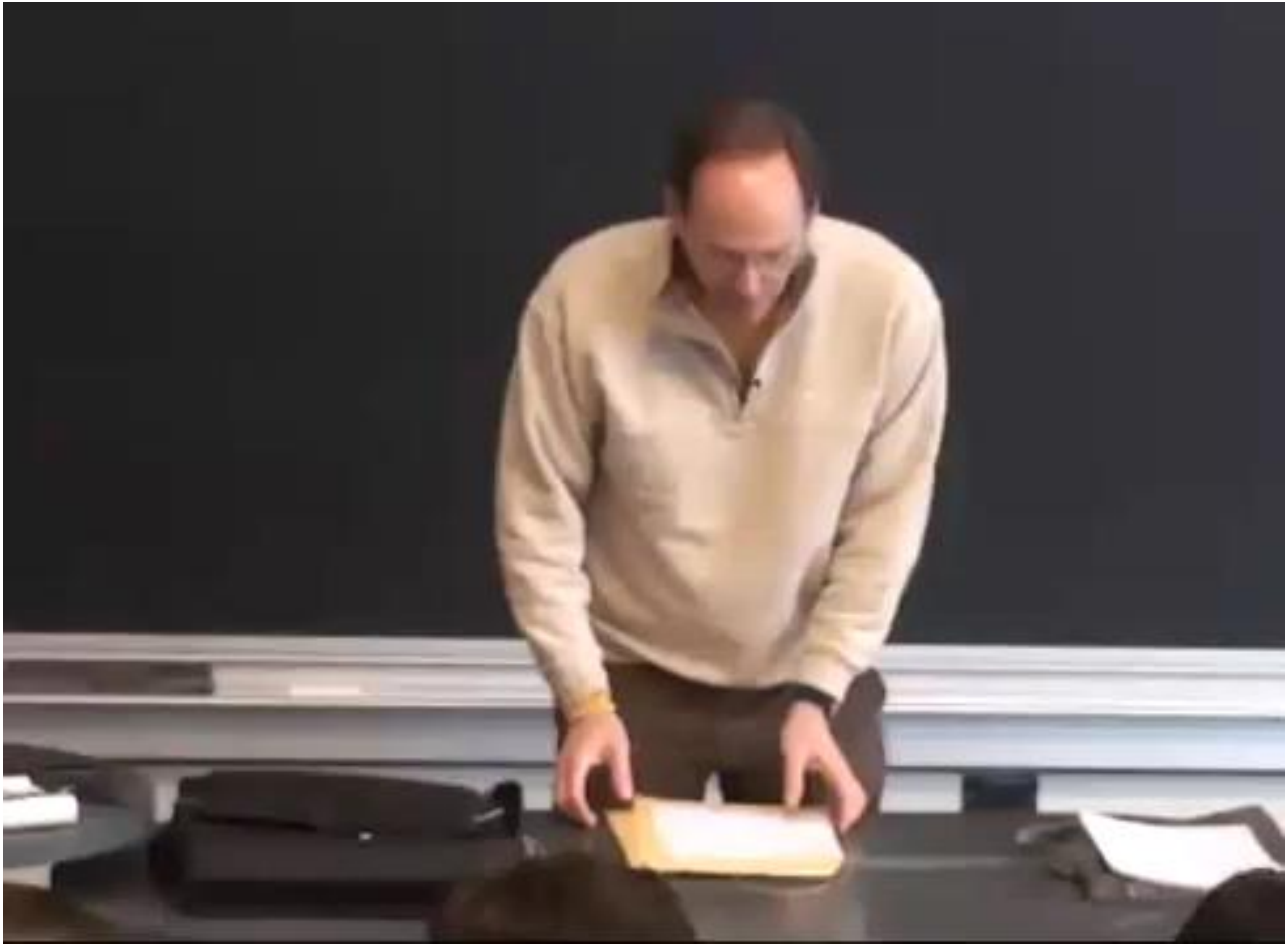
- The role of eigenvalues
- How bifurcations in 2D are different from 1D
- The role of slow dynamics near the bifurcation onsets

Hopf bifurcation for self-watch

Lecture 13:

https://www.youtube.com/watch?v=qV0tCroLOHk&list=PLbN57C5Zdl6j_qJA-pARJnKsmROzPnO9V&index=13

Till ~ 58:00



Main points to pay attention to:

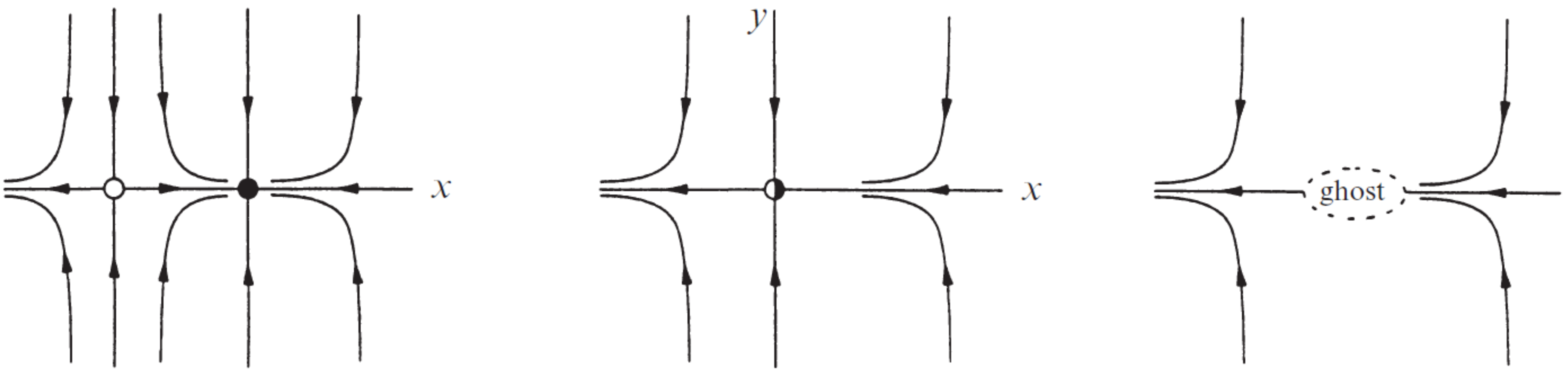
- Onset of oscillations
- Super- and sub-critical bifurcations
- Why is it important to understand the amplitude of the oscillation?

BIFURCATIONS REVISITED

Saddle-Node Bifurcation

$$\dot{x} = \mu - x^2$$

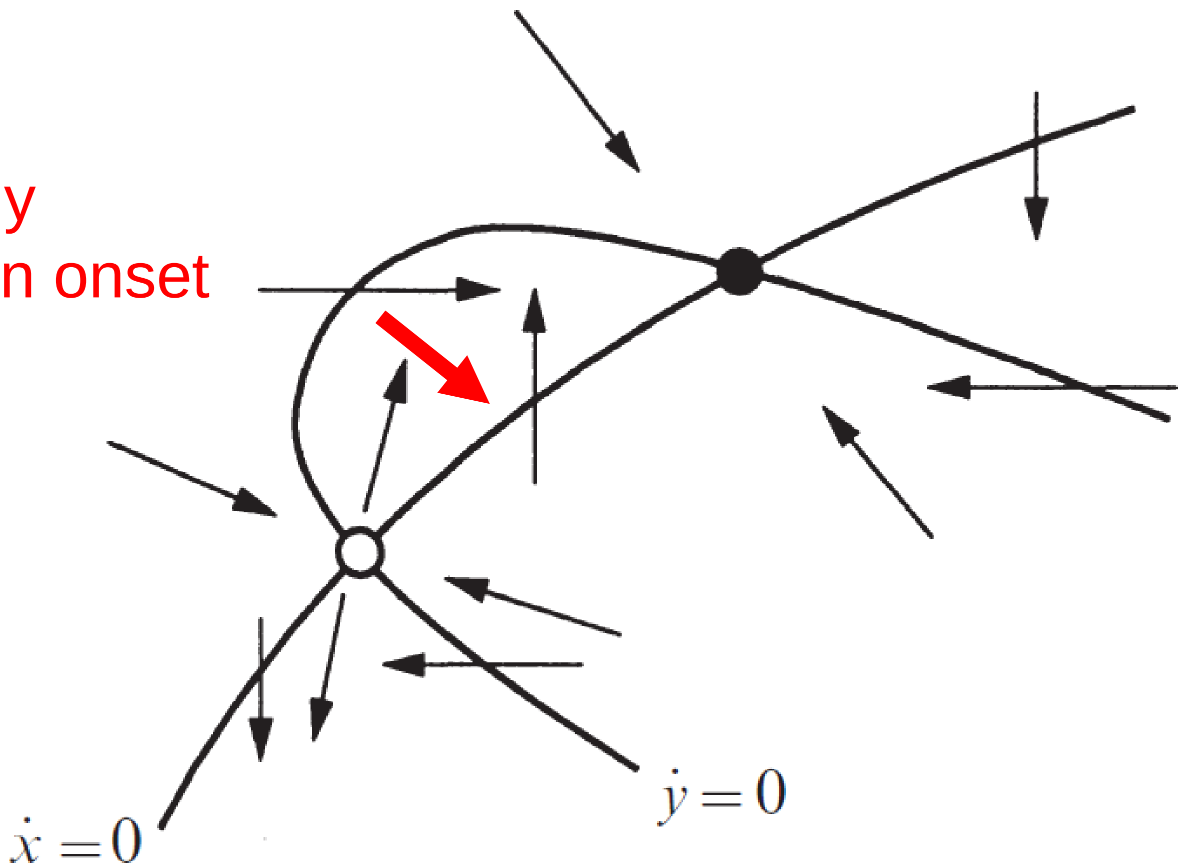
$$\dot{y} = -y.$$



$\mu > 0$ $\mu = 0$ $\mu < 0$

Saddle-Node Bifurcation

nullclines become locally *tangent* at the bifurcation onset



Does the dynamics in y axis really matters?

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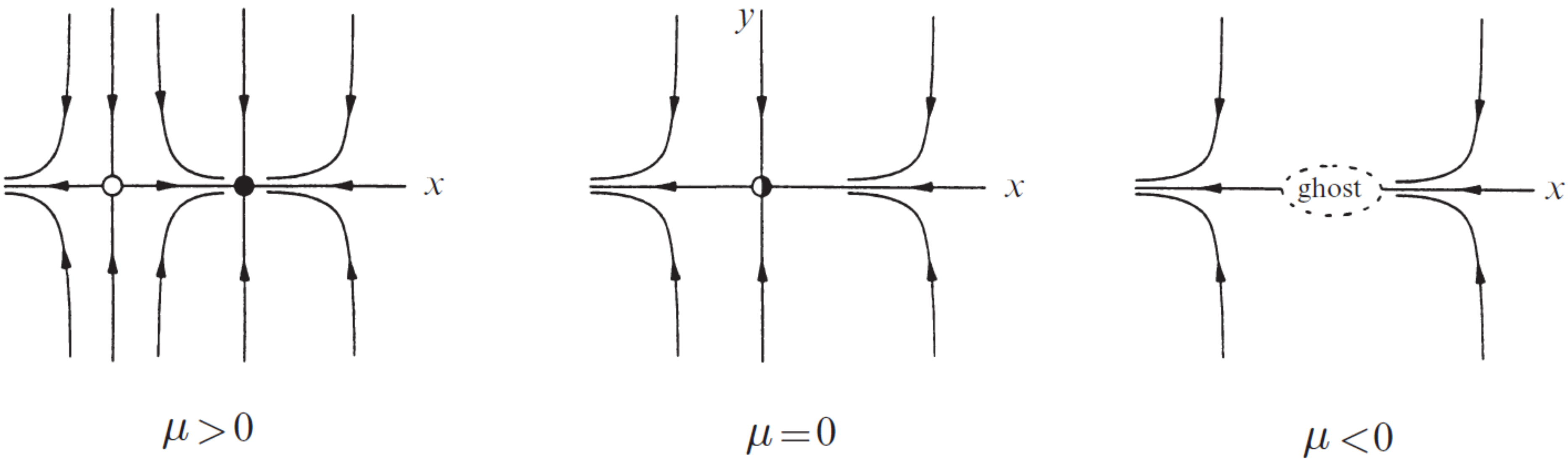
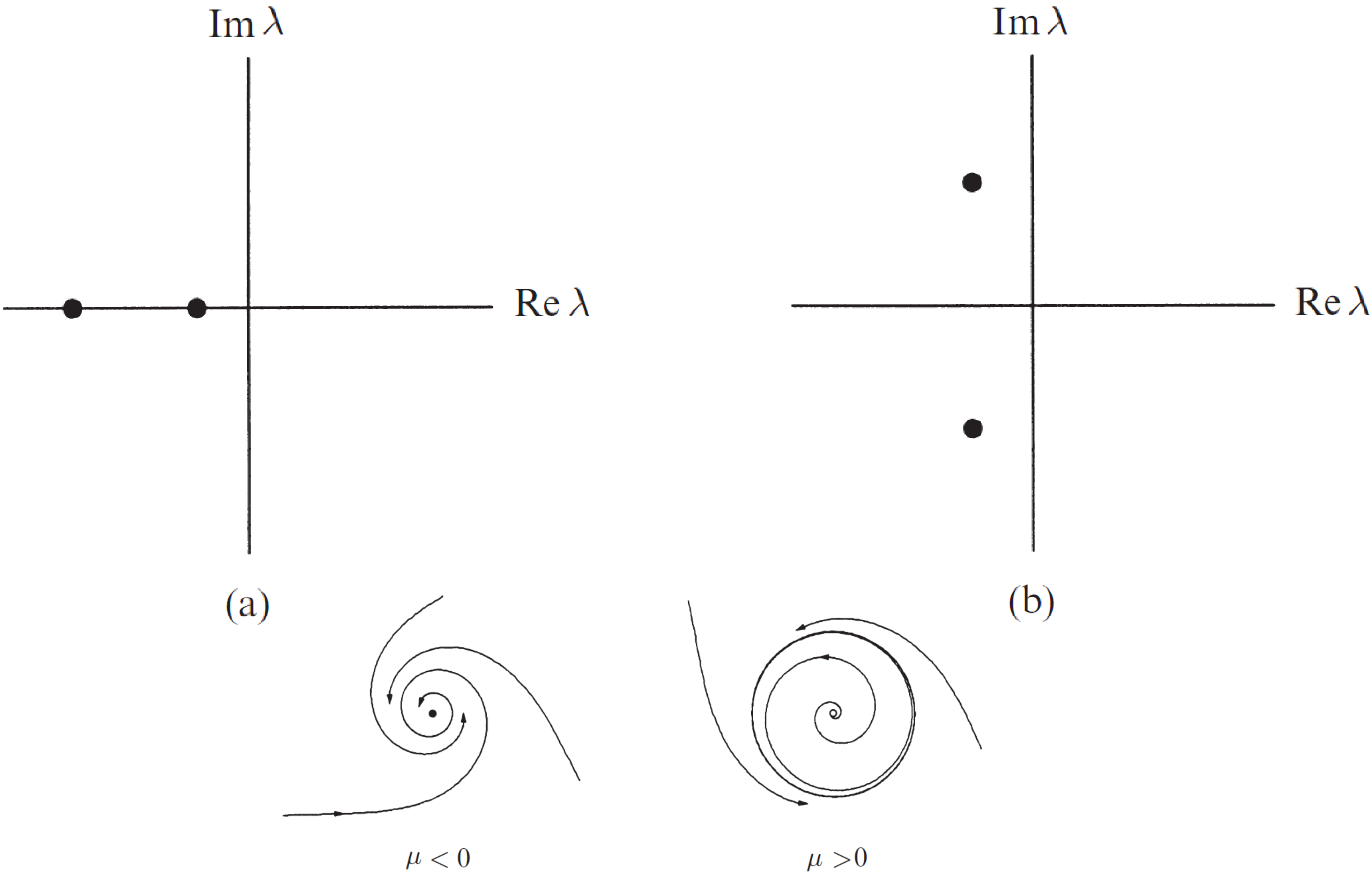
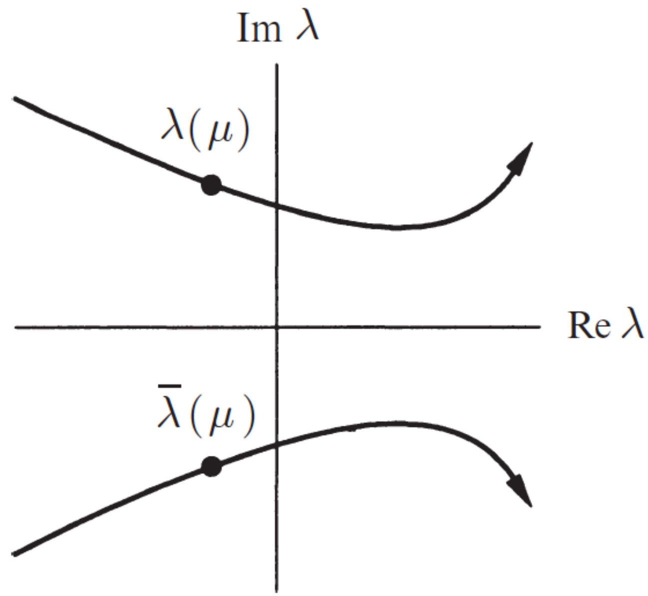
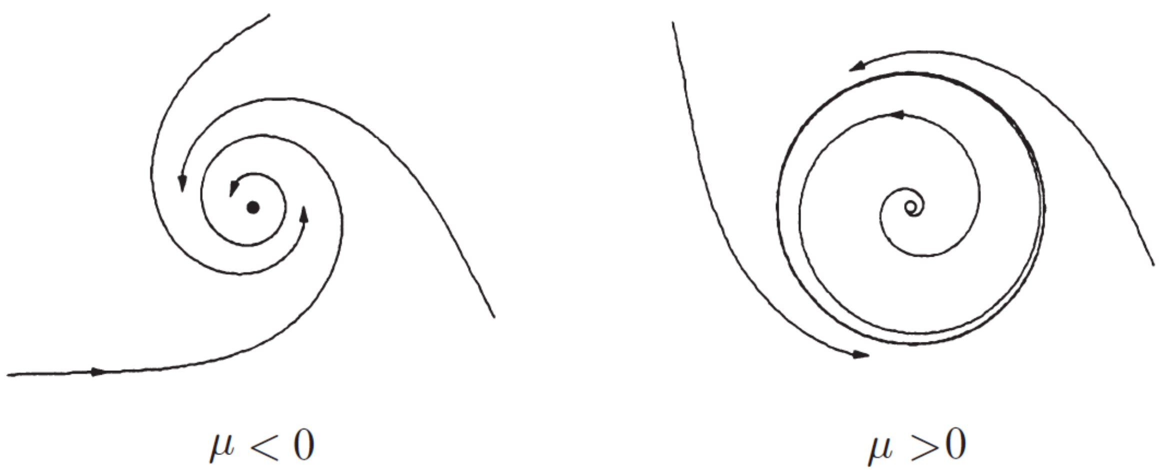


Hopf Bifurcations

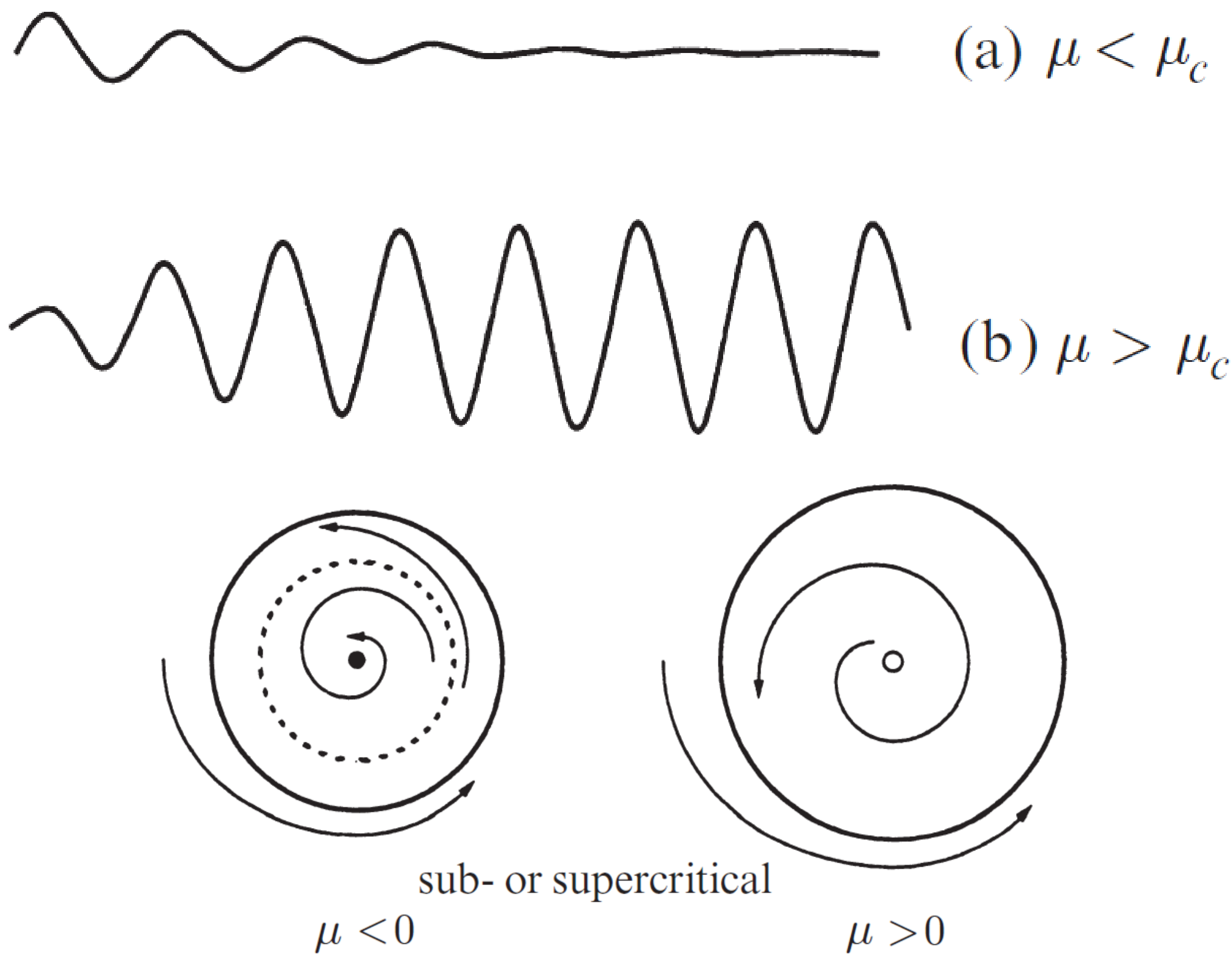


Hopf Bifurcations



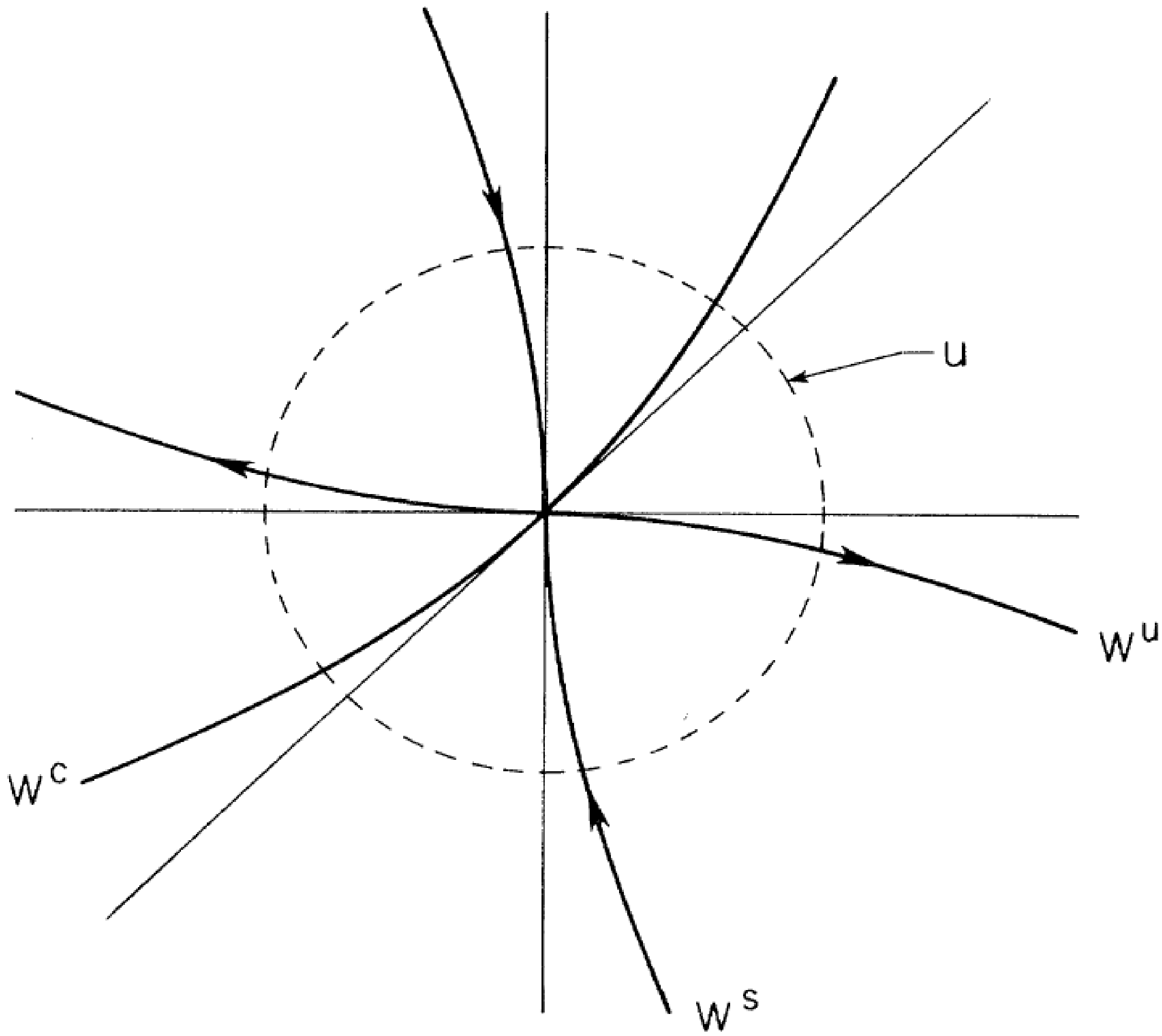


Hopf Bifurcations

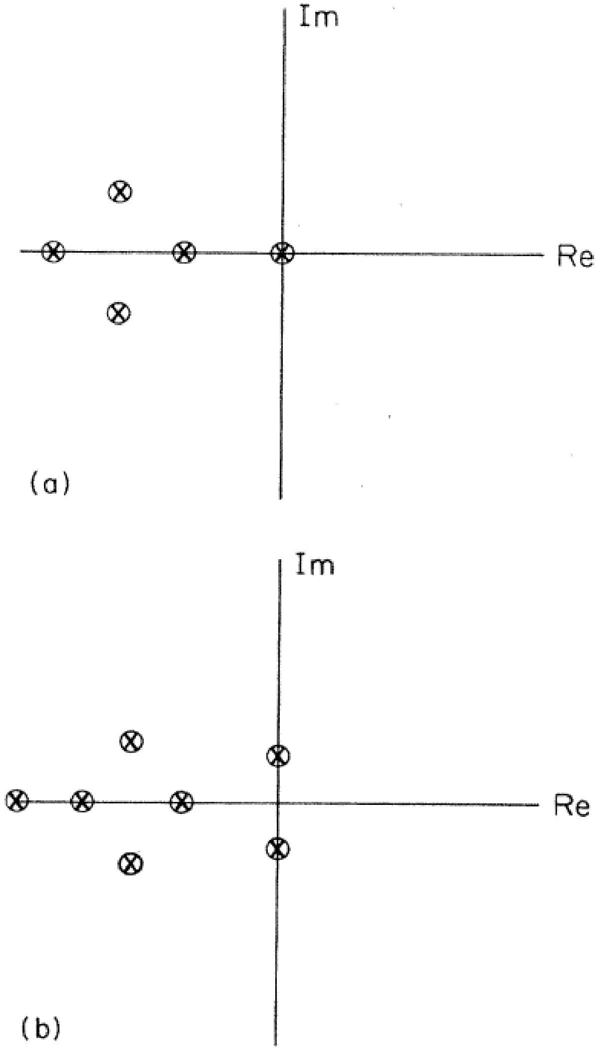


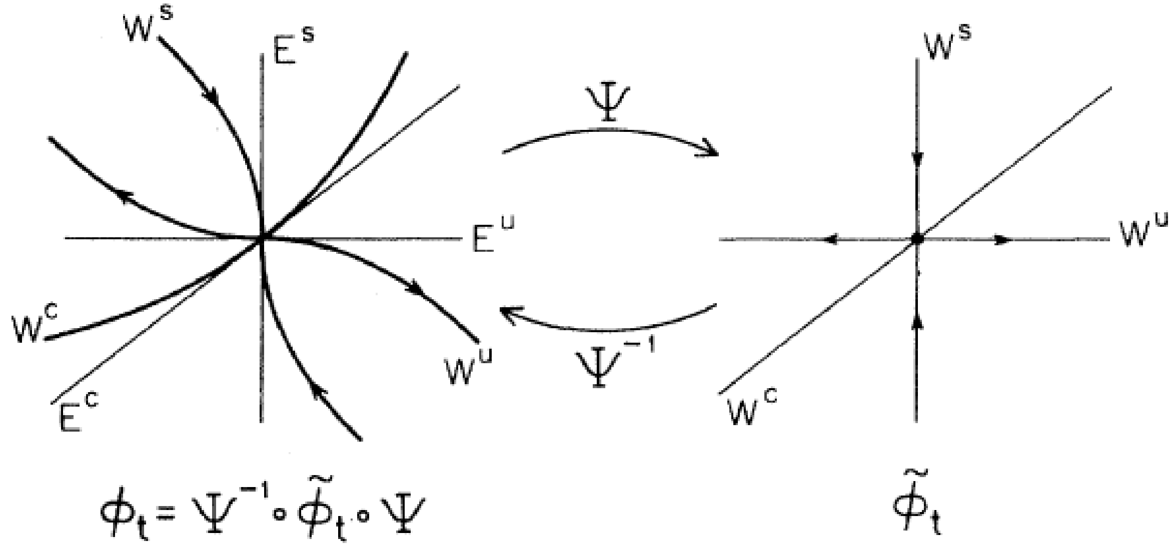
Slow dynamics, enslaving and center manifold reduction

J.D. Crawford, Introduction to bifurcation theory, RPM **63**, 991 (1991)



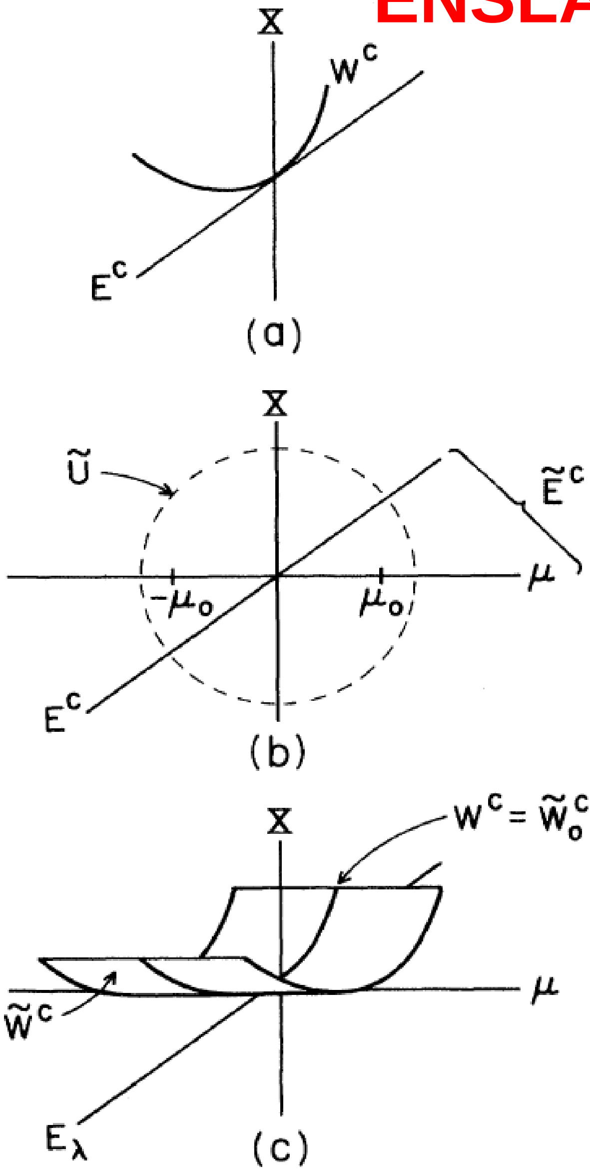
Weakly nonlinear analysis

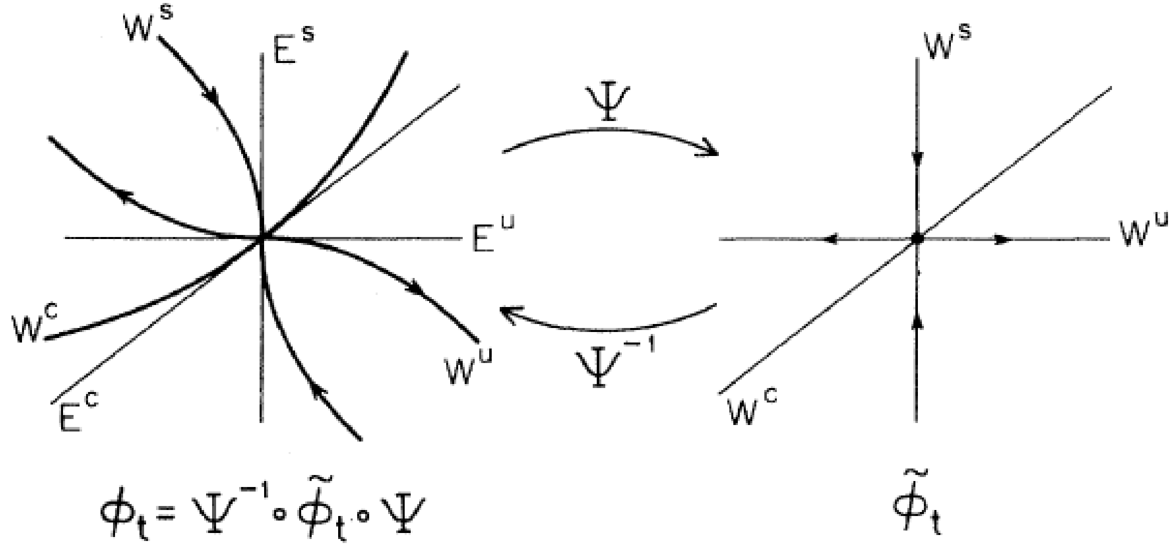




What is so special about the onset?

ENSLAVING by the critical modes





Center-manifold reduction and normal-form theory

- Bifurcation analysis of an arbitrary high dimensional system can be reduced to a simple **normal form**

- The reduction in dimensionality is accomplished by observing that the interesting dynamics near a bifurcation occurs on a low-dimensional subset of phase space called the **center manifold**

- The dimension of this center manifold determines the dimension of the normal form

Center-manifold reduction and normal-form theory

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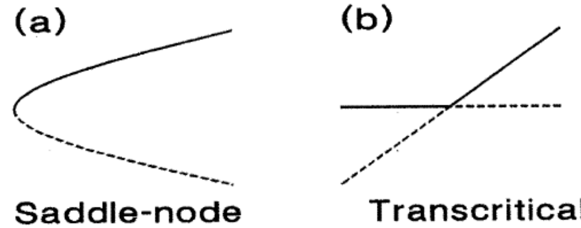
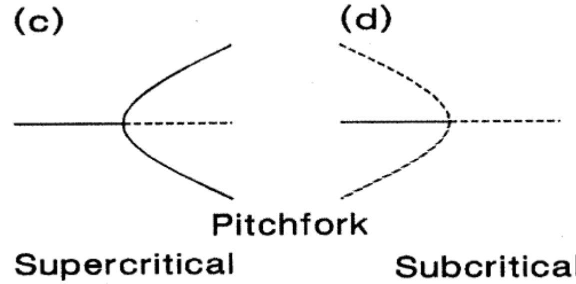
(4) Study the bifurcations described by the unfolded normal form. In this analysis, one truncates the unfolded system at some order and considers the resulting system. Once the truncated system is understood, the effect of restoring the higher-order terms can be discussed.[9]

$$\partial_t u = R - u^2$$



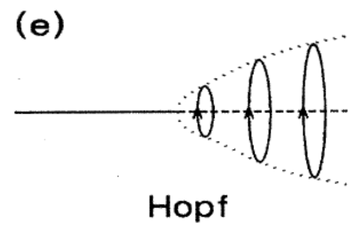
$$\partial_t u = Ru - u^2$$

$$\partial_t u = Ru - gu^3$$



$$z = u_1 + i\, u_2$$

$$\partial_t z = R\, z + |z|^2\, z$$



$$\partial_t u_1 = -\; u_2 + Ru_1 - (u_1^2 + u_2^2)\, u_1\ ,$$

$$\partial_t u_2 = \quad u_1 + Ru_2 - (u_1^2 + u_2^2)\, u_2\ .$$

$$\partial_t u = h + Ru + pu^2 - g\ \ u^3$$