Suggested home exercises – lecture 8

8.4.3 (Homoclinic bifurcation) Using numerical integration, find the value of μ at which the system $\dot{x} = \mu x + y - x^2$, $\dot{y} = -x + \mu y + 2x^2$ undergoes a homoclinic bifurcation. Sketch the phase portrait just above and below the bifurcation.

8.6.1 ("Oscillator death" and bifurcations on a torus) In a paper on systems of neural oscillators, Ermentrout and Kopell (1990) illustrated the notion of "oscillator death" with the following model:

$$\dot{\theta}_1 = \omega_1 + \sin\theta_1 \cos\theta_2, \qquad \dot{\theta}_2 = \omega_2 + \sin\theta_2 \cos\theta_1,$$

where $\omega_1, \omega_2 \geq 0$.

a) Sketch all the qualitatively different phase portraits that arise as ω_1, ω_2 vary.
b) Find the curves in ω_1, ω_2 parameter space along which bifurcations occur, and classify the various bifurcations.
c) Plot the stability diagram in ω_1, ω_2 parameter space.

8.7.9 Consider the vector field given in polar coordinates by $\dot{r} = r - r^2$, $\dot{\theta} = 1$.
a) Compute the Poincaré map from S to itself, where S is the positive x-axis.
b) Show that the system has a unique periodic orbit and classify its stability.
c) Find the characteristic multiplier for the periodic orbit.

8.7.10 Explain how to find Floquet multipliers numerically, starting from perturbations along the coordinate directions.