

**Page of formulas and values for important constants for the Physics 3A Exam**

De Broglie Wavelength:  $\lambda = h/p$ . Wavenumber:  $k = 2\pi/\lambda$ . Momentum:  $p = mv = \hbar k$ .

$\hbar = h/2\pi$ . De Broglie frequency:  $\nu = E/h$ . Kinetic energy:  $E = \frac{p^2}{2m} = \frac{1}{2}mv^2$ . De Broglie phase velocity:  $c = \lambda\nu$ .

TISE:  $\hat{H}\psi_n = E_n\psi_n$ . Hamiltonian:  $\hat{H} = \frac{\hat{p}^2}{2m} + V(x) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)$ .

Momentum:  $\hat{p}_x = -i\hbar\frac{\partial}{\partial x}$ .  $\hat{\mathbf{p}} = -i\hbar\nabla$ .

TDSE:  $i\hbar\frac{\partial\Psi(x,t)}{\partial t} = \hat{H}\Psi(x,t)$ .

Born interpretation:  $p(x,t) = \Psi^*(x,t)\Psi(x,t) = |\Psi(x,t)|^2$ .

Expectation value:  $\langle A \rangle = \int_{-\infty}^{\infty} \Psi^*(x,t) \hat{A}\Psi(x,t) dx$ . For a general probability density:

$\langle A \rangle = \int_{-\infty}^{\infty} p(x,t) A(x) dx$ . For discrete variable:  $\langle A \rangle = \sum_i P_i A_i$ .

Commutator:  $[\hat{A}, \hat{B}] \equiv \hat{A}\hat{B} - \hat{B}\hat{A}$ . Standard deviation (uncertainty):  $\Delta A = \sqrt{\langle A^2 \rangle - \langle A \rangle^2}$ .

Uncertainty Relation:  $\Delta A \Delta B \geq \frac{1}{2} |\langle [\hat{A}, \hat{B}] \rangle|$ .

Position-momentum commutator:  $[\hat{x}, \hat{p}] = i\hbar \rightarrow \Delta x \Delta p \geq \frac{\hbar}{2}$ .

TISE for harmonic oscillator:  $\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \left(\frac{kx^2}{2}\right)\right)\psi(x) = E_n\psi(x)$

Energy levels of harmonic oscillator:  $E_n = \left(n + \frac{1}{2}\right)\sqrt{\frac{k}{m}}\hbar = \hbar\omega\left(n + \frac{1}{2}\right)$ . Harmonic oscillator

raising and lowering operators:  $\hat{a}^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} - \frac{i}{m\omega}\hat{p}\right)$ ;  $\hat{a} = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} + \frac{i}{m\omega}\hat{p}\right)$ .

$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$ .  $\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$ .

Dirac notation:  $\langle n|m\rangle = \int_{-\infty}^{\infty} \psi_n^*(x)\psi_m(x)dx = \delta_{nm}$ .  $\langle n|\hat{A}|m\rangle = \int_{-\infty}^{\infty} \psi_n^*(x)\hat{A}\psi_m(x)dx$ .

Angular momentum:  $\hat{L}_x = -i\hbar\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)$ ,  $\hat{L}_y = -i\hbar\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right)$ ,  $\hat{L}_z = -i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)$ .

$$[\hat{L}_x, \hat{L}_y] = i\hbar\hat{L}_z; [\hat{L}_y, \hat{L}_z] = i\hbar\hat{L}_x; [\hat{L}_z, \hat{L}_x] = i\hbar\hat{L}_y;$$

Angular momentum in spherical coordinates:  $\hat{L}_x = i\hbar\left(\sin(\phi)\frac{\partial}{\partial\theta} + \cot(\theta)\cos(\phi)\frac{\partial}{\partial\phi}\right)$ ,  $\hat{L}_y = i\hbar\left(-\cos(\phi)\frac{\partial}{\partial\theta} + \cot(\theta)\sin(\phi)\frac{\partial}{\partial\phi}\right)$ ,  $\hat{L}_z = -i\hbar\frac{\partial}{\partial\phi}$ .

$$\hat{L}^2 = -\hbar^2 \left( \frac{1}{\sin(\theta)} \frac{\partial}{\partial \theta} \left( \sin(\theta) \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin^2(\theta)} \frac{\partial^2}{\partial \phi^2} \right). \hat{L}_{\pm} = \hat{L}_x \pm i\hat{L}_y;$$

$$\hat{L}_{\pm}|l, m_l\rangle = \hbar\sqrt{l(l+1) - m_l(m_l \pm 1)}|l, m_l \pm 1\rangle$$

Laplacian in spherical coordinates:

$$\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2}$$

$$\text{Hydrogen atom: } \psi_{nlm}(\vec{r}) = R_{nl}(r)Y_{lm}(\theta, \phi); \hat{H}\psi_{nlm}(\vec{r}) = -\frac{\mu Z^2 e^4}{2\hbar^2 n^2} \psi_{nlm}(\vec{r}); \mu = \frac{m_e}{1 + \frac{m_e}{M}}$$

Orbital magnetic moment:  $\mu_{orb,z} = -m_l \mu_B$ . Magnetic dipole interaction energy:  $V_B = -\vec{\mu} \cdot \vec{B}$ .

Volume element in cartesian coordinates:  $dx dy dz$ .

Volume element in spherical coordinates:  $r^2 \sin \theta dr d\theta d\phi$ .

Boltzmann distribution:  $f_B(\epsilon) = \frac{1}{Z} e^{-\frac{\epsilon}{k_B T}}$ ;  $Z = \int e^{-\frac{\epsilon}{k_B T}}$  the integral is over all possible states

Fermi Dirac distribution:  $f_{FD}(\epsilon) = \frac{1}{1 + e^{\frac{\epsilon - \mu}{k_B T}}}$ ; Chemical potential:  $\mu = \mu(T)$ .  $\mu(T = 0) = \epsilon_F$ .

Reflection and Transmission coefficients:  $R = \frac{j_{ref}}{j_{inc}} = \frac{|\psi_{ref}|^2}{|\psi_{inc}|^2}$ ,  $T = \frac{j_{trans}}{j_{inc}} = \frac{k_{trans} |\psi_{trans}|^2}{k_{inc} |\psi_{inc}|^2}$ ,

$$R + T = 1.$$

Charge density:  $\rho = q|\psi|^2$ . Velocity:  $\langle \vec{v} \rangle = \frac{\langle \vec{p} \rangle}{m} = \frac{\hbar \langle \vec{k} \rangle}{m}$ . Current density:  $\vec{j} = nq\langle \vec{v} \rangle = \rho \langle \vec{v} \rangle$

### **Trigonometric identities:**

$$\sin^2(\theta) = \frac{1}{2}(1 - \cos(2\theta)); \sin(2\theta) = 2 \sin(\theta) \cos(\theta); \cos(2\theta) = \cos^2(\theta) - \sin^2(\theta);$$

$$e^{i\theta} = \cos(\theta) + i\sin(\theta)$$

### **Constants:**

Electron mass:  $9.11 \times 10^{-31} kg$ . Proton mass:  $1.673 \times 10^{-27} kg$ . Electron charge:  $e = 1.6 \times 10^{-19} C$ . Electron volt:  $1eV = 1.6 \times 10^{-19} J$ . Planck constant:  $h = 6.63 \times 10^{-34} J \cdot s$ .  $\hbar = 1.054 \times 10^{-34} J \cdot s$ . Speed of light in vacuum:  $c = 3 \times 10^8 m/s$ . Rest energy of electron:  $m_e c^2 = 0.511 \times 10^6 eV$ . Bohr magneton:  $9.274 \times 10^{-24} J \cdot T^{-1}$ . Boltzmann constant:  $k_B = 1.38 \times 10^{-23} J \cdot K^{-1} = 8.62 \times 10^{-5} eV \cdot K^{-1}$ . Avogadro number:  $N_A = 6.022 \times 10^{23}$ .

### **Useful integrals:**

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}, \int_{-\infty}^{\infty} e^{-ax^2+bx} dx = \sqrt{\pi/ae} e^{\frac{b^2}{4a}}, \int_0^{\infty} \sqrt{x} e^{-nx} dx = \frac{\sqrt{\pi}}{2n^{3/2}},$$

$$\int_0^{\infty} x^n e^{-x} dx = \Gamma(n+1) \text{ for } n > -1; \text{ If } n \in \text{Integers and } n \geq 1 \Gamma(n) = n - 1!$$

$$\int_0^{2\pi} \cos^2(x) dx = \pi, \int \sin^2(x) x dx = \frac{x^2}{4} - \frac{1}{8} \cos(2x) - \frac{1}{4} x \sin(2x).$$

### **Fourier transform:**

$$\mathcal{F}(f(x)) = \tilde{f}(k) = \int_{-\infty}^{\infty} f(x) e^{ikx} dx \quad \mathcal{F}^{-1}(\tilde{f}(k)) = f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{f}(k) e^{-ikx} dk$$

$$\mathcal{F}\left(\frac{df(x)}{dx}\right) = -ik\tilde{f}(k)$$

$$\text{Dirac's delta function: } \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i(k-k')x} dx = \delta(k-k') \quad \int_{-\infty}^{\infty} f(x) \delta(x-a) dx = f(a)$$

### **Electromagnetic field:**

Force due to electric field:  $\vec{F} = q\vec{E}$ . Lorentz force:  $\vec{F} = \frac{q}{c} \vec{v} \times \vec{H}$ . Energy density in vacuum:

$$u = \frac{1}{2} (\epsilon_0 \vec{E}^2 + \mu_0 \vec{H}^2). \text{ Ohm's law: } \vec{j} = \underline{\underline{\sigma}} \cdot \vec{E}, \text{ conductivity tensor: } \underline{\underline{\sigma}} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}.$$

### **Heat capacity:**

$$c_V = \left(\frac{\partial u}{\partial T}\right)_V, u = \frac{U}{V} \quad U \text{ is the internal energy and } V \text{ the volume}$$

### **Density of states for "electron gas":**

$$1D: g(\epsilon) = \frac{n}{2} \frac{1}{\sqrt{\epsilon \epsilon_F}}$$

$$3D: g(\epsilon) = \frac{3n}{2\epsilon_F} \sqrt{\frac{\epsilon}{\epsilon_F}}$$