Hydrogen atom - Superposition of States

exercise 3_4936

Let $\varphi_{n\ell m}(\boldsymbol{r})$ denote the properly-normalized energy eigenfunctions of the Hydrogen atom (H-atom) with principle quantum number n and angular momentum quantum numbers ℓ and m. Consider an electron in the following state which is a superposition of H-atom eigenstates:

$$\psi(\boldsymbol{r}) = C\,[\varphi_{100}(\boldsymbol{r}) + 4i\varphi_{210}(\boldsymbol{r}) - 2\sqrt{2}\,\varphi_{21-1}(\boldsymbol{r})]$$

(a) Find the normalization constant C. Can C be complex, real or imaginary?

(b) What is the expectation value of the Hamiltonian for the H-atom in this superposition of states? Calculate your answer in units of eV.

In parts (c) and (d), find the following expectation values:

(c) Find $\langle \boldsymbol{L}^2 \rangle$.

(d) Find $\langle L_z \rangle$.

(e) What is the probability of finding the electron in the $\varphi_{210}(\boldsymbol{r})$ state?

(f) If the electron is in the $\varphi_{100}(\boldsymbol{r})$ state, find the radius at which the radial probability density is a maximum. How does this compare with the Bohr radius? What is the significance?

(g) Find the wavefunction $\psi(\boldsymbol{r}, t)$ at some later time t.
