5 Detailed balance

The principle of detailed balance states that at thermodynamical equilibrium, each elementary process should be equilibrated by its reverse process. According to the general principles of quantum mechanics (time reversibility, etc.), the direct and reverse transitionb rates between two simple quantum states are equal. Generally the final energy state could be degenerate with respect to the rest of the quantum numbers (azimuthal, spin, etc.); then the ratio of the transition rates for the direct and reverse processes is equal to the ratio of the statistical weights of the states:

$$\frac{W_{1\to 2}}{W_{2\to 1}} = \frac{g_2}{g_1}. \tag{5.1}$$

This relation directly leads to the detailed balance.

For example, the reaction (4.5) is reverse to the reaction $n \to p + e^- + \tilde{\nu}$. The neutron decay rate is $W_{n\to p} = \frac{1}{880}\ \mathrm{s}^{-1}$. According to the principle of detailed balance,

$$\frac{W_{p\to n}}{W_{n\to p}} = \frac{g_{e^-,\tilde{\nu}}}{g_{e^+,\nu}}, \tag{5.2}$$

where $g_{e^-,\tilde{\nu}}$ and $g_{e^+,\nu}$ are statistical weights of free leptons produced in the reactions. Inasmuch as the leptons in two reactions are produced with roughly the same energy, ~ 1 MeV, the statistical weights are roughly equal so that one can substitute $W_{p\to n}$ by $W_{n\to p}$ in the reaction rate (4.17).

5.1 Resonant absorption

Consider a system in a quasi-equilibrium state with the energy E_0 and the decay rate Γ, the last being a sum of the decay rates in different channels. To be definite, let it be an excited nucleus, one of the decay channels being the emission of a free neutron, $(A, Z) \to (A-1, Z) + n$, with the rate Γ_n. The energy distribution of the emitted neutrons is found as follows.

In the quasi-equilibrium state, the time dependence of the wave function is $\Psi \propto \exp\left[-\left(\frac{E_0}{\hbar} i + \frac{\Gamma}{2}\right) t\right]$. The energy distribution is obtained by expansion of the wave function in the proper states of the energy operator,

$$\Psi(t) = \int a(E) e^{-i\frac{E}{\hbar}t} dE. \tag{5.3}$$

The reverse Fourier transform yields

$$a(E) = \frac{C}{E - E_0 + i\frac{\hbar\Gamma}{2}}. \tag{5.4}$$

The probability for the nucleus to emit a neutron with the energy E is $\rho(E) \propto |a(E)|^2$. The normalization is found from the condition that $\int \rho(E)dE = 1$. Then one gets a Lorentzian distribution

$$\rho(E)dE = \frac{\hbar\Gamma}{(E-E_0)^2 + (\hbar\Gamma/2)^2}\frac{dE}{2\pi}. \tag{5.5}$$

The reverse process is the resonant absorption of neutrons; the cross section is found making use of the results of the detailed balance principle. Namely, we consider the reaction $(A, Z) \leftrightarrow (A-1, Z) + n$. According to the detailed balance principle, the rates of the direct and reverse reactions are related by eq. (5.1) The rate of the direct reaction is $W_{1\to 2} = \Gamma_n \rho(E) dE$; the statistical weight of the final state is presented, neglecting the spin contribution, as $g_2 = 4\pi p^2 dp V$, where p is the momentum of the free neutron. The rate of the neutron absorption is written as $P_{2\to 1} = \sigma v / V$. Neglecting again the spin contribution to the statistical weights, we take $g_1 = 1$. Then eq. (5.1) is written as

$$\frac{\rho(E) dE V}{\sigma v} = \frac{4\pi p^2 dp V}{(2\pi\hbar)^3}. \tag{5.6}$$

Now taking into account that $dE = v dp$, one gets finally

$$\sigma = \pi \lambda\!\!\!^{-2} \frac{\hbar^2 \Gamma_n \Gamma}{(E - E_0)^2 + (\hbar\Gamma/2)^2}, \tag{5.7}$$

where $\lambda\!\!\!^{-} = \hbar/p$ is the neutron de Broglie wavelength. This expression is known as the Breit-Wigner formula.

The same consideration could be applied to radiative transitions between atom's levels. In this case, Γ^{-1} is the life time of the excited level (which is determined by radiation and collisional transitions onto all levels), Γ_n the Einstein coefficient for the transition. In terms of the radiation frequencies, the absorption cross-section is now written as

$$\sigma = \pi \lambda\!\!\!^{-2} \frac{A \Delta\omega}{(\omega - \omega_0)^2 + (\Delta\omega/2)^2}. \tag{5.8}$$

Here Γ was substituted by the line width, $\Delta\omega$. In the center of the line, the cross section is estimated as

$$\sigma_{\max} \sim 4\pi \lambda\!\!\!^{-2} \frac{A}{\Delta\omega}. \tag{5.9}$$

Specifically if the line broadening is determined only by radiation transitions (natural broadening), $\Delta\omega \sim A$ so that $\sigma_{max} \sim \lambda^2$. Substituting the estimate (3.4) for A, one gets

$$\sigma_{max} \sim 10 \frac{r_e c}{\Delta\omega} \frac{E}{\hbar\omega}. \tag{5.10}$$

Here E is the electron energy at the upper level.