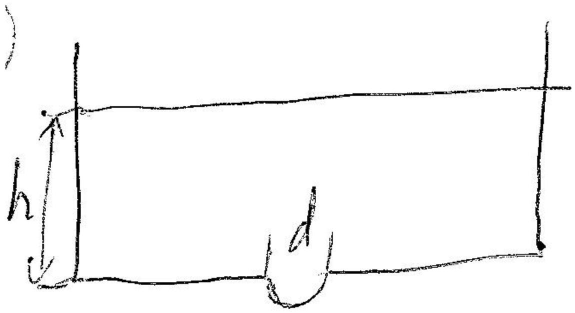
1. **In the bottom of a vessel with mercury there is a round hole of diameter 70 μm. At what maximum thickness of the mercury layer will the liquid still not flow out through this hole? The surface tension of mercury is 0.5 N/m.**



The excess hydrostatic pressure at the bottom of the vessel is $p = \rho g h$. The surface tension provides the excess pressure within the hole as given by the Laplace formula

$$p = \frac{2\gamma}{R},$$

where R is the curvature radius of the surface within the hole. The equilibrium condition:

$$\rho g h = \frac{2\gamma}{R} .$$

$$h = \frac{2\gamma}{\rho g R} = \frac{4\gamma}{\rho g d} = \frac{4 \cdot 0.5}{13.6 \cdot 10^3 \cdot 10 \cdot 70 \cdot 10^{-6}} = 0.2 \text{ m}$$

2. **Find the difference in height of mercury columns in two communicating vertical capillaries whose diameters are d_1=0.5 mm and d_2=1 mm, if the contact angle θ=138°, the surface tension is 0.5 N/m.**

The curvature radius of the meniscus is

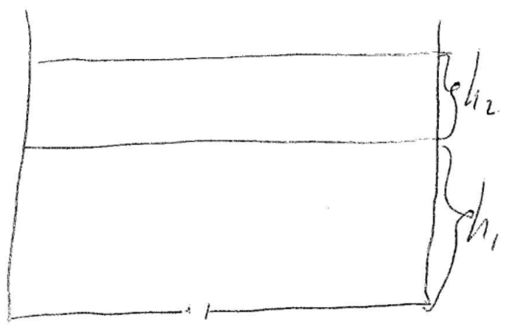
$$R = \frac{d}{2\sin(\theta - 90^o)} = \frac{d}{2\cdot 0.74} = \frac{d}{1.5}$$

The pressure equilibrium condition:

$$\rho g h + \frac{2\gamma}{R_2} = \frac{2\gamma}{R_1}$$

$$h = \frac{2\gamma \cdot 1.5}{\rho g}\left(\frac{1}{d_1} - \frac{1}{d_2}\right)$$=1.1 cm

3. **A wide vessel with a small hole in the bottom is filled with water and kerosene. Find the velocity of the water flow, if the thickness of the water layer is equal to h_1=30 cm and that of the kerosene layer to h_2 = 20 cm. Density of kerosene is 0.8 g/cm^3.**



The pressure at the kerosene-water boundary is $p = p_{\text{atm}} + \rho_k g h_2$. Bernoulli eq. for water flow:

$$\frac{p}{\rho_w} + g h_1 = \frac{p_{\text{atm}}}{\rho_w} + \tfrac{1}{2}\text{v}^2$$

$$\text{v} = \sqrt{2\left(\frac{p - p_{\text{atm}}}{\rho_w} + g h_1\right)} = \sqrt{2g\left(\frac{\rho_k}{\rho_w} h_2 + h_1\right)}$$

4. **Through a pipe of diameter *D*, water is pumped from a lake to the water-tower. (a) If the rate of water supply (volume per unit time) is *Q*, what is the speed of the water in the pipe? (b) What pressure is necessary to deliver this flow? (c) What is the power of the pump? The height of the water-tower is H.**

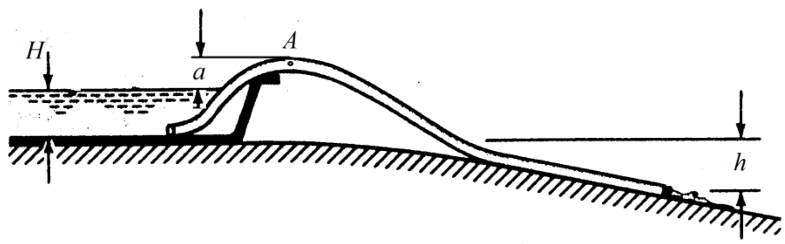
$$\text{v}=\frac{Q}{S} = \frac{4Q}{\pi D^2}$$

$$p = \rho g h + \tfrac{1}{2}\rho \text{v}^2$$

A mass of water *dm* acquires the energy $dE = dm\left(\frac{1}{2}\mathrm{v}^2 + gh\right)$. The required power is

$$N = \frac{dE}{dt} = \frac{dm}{dt}\left(\frac{1}{2}\mathrm{v}^2 + gh\right) = \rho Q\left(\frac{1}{2}\mathrm{v}^2 + gh\right)$$

5. על מנת לרוקן בריכת שכשוך, משתמשים בצינור גינה פלסטי עם קוטר פנימי d כמתואר באיור. מצא\י את קצב הזרימה (נפח ליחידת זמן) בצינור ולחץ בנקודה A. אפשר להזניח את הצמיגות.



The Bernoulli equation for a flow line connecting the surface and the hose nozzle:

$$p_{atm} + \rho g H = p_{atm} - \rho g h + \frac{1}{2}\rho \mathrm{v}^2$$

$$\mathrm{v} = \sqrt{2g(H+h)}$$

The flow rate $Q = \mathrm{v}S = \frac{\pi}{4}d^2\mathrm{v} = \frac{\pi}{4}d^2\sqrt{2g(H+h)}$

In order to find the pressure at the point A, write the Bernoulli equation for the surface and the point A

$$p_{atm} + \rho g H = p_A + \rho g(H+a) + \frac{1}{2}\rho \mathrm{v}^2$$

The continuity equation, *Sv=const,* implies that the velocity within the hose does not vary because the cross section is constant. Then

$$p_A = p_{atm} - \rho g a - \frac{1}{2}\rho \mathrm{v}^2 = p_{atm} - \rho g a - \frac{1}{2}\rho \cdot 2g(H+h) = p_{atm} - \rho g(a+h+H)$$

Note that $p_A < p_{atm}$.

6. הקוטר של זרבובית של הצינור לכיבוי אש הוא *d=4*cm ואילו הקוטר של הצינור עצמו הוא *D=8*cm. איזו עודף לחץ דרוש בתוך הצינור על מנת לספק קצב זרימה *q*=20litr/s? אפשר להזניח את הצמיגות.

q=Sv.

$$\mathrm{v}_1 = \frac{4q}{\pi D^2} \; ; \; \mathrm{v}_2 = \frac{4q}{\pi d^2}$$

$$p + \frac{\rho \mathrm{v}_1^2}{2} = p_{\mathrm{atm}} + \frac{\rho \mathrm{v}_2^2}{2}$$

$$p - p_{atm} = \frac{1}{2}\rho\left[\left(\frac{4q}{\pi d^2}\right)^2 - \left(\frac{4q}{\pi D^2}\right)^2\right] = \frac{8\rho q^2}{\pi^2 d^2}\left[1 - \left(\frac{d}{D}\right)^2\right]$$

$$= \frac{8 \cdot \left(2 \cdot 10^4\right)^2}{\pi^2 4^4}\left(1 - \frac{1}{2^4}\right) \approx \frac{10^8}{10 \cdot 8} \approx 10^6 \text{ dyne/cm}^2 = 100\text{kPa}$$