1. **A long cylinder of radius R_1 is displaced along its axis with a constant velocity v_0 inside a stationary co-axial cylinder of radius R_2. The space between the cylinders is filled with liquid with the viscosity coefficient η. Find the velocity of the liquid as a function of the distance from the axis of the cylinders and the friction force per unit length of the inner cylinder.**

   Let us consider a cylindrical liquid layer of radius *r* and thickness *dr* (such that $R_1<r<R_2$). The equilibrium condition implies that the viscous forces on the inner and outer boundaries of the layer cancel each other. The viscous force (per unit length) on the inner boundary is

$$\left[\eta \frac{d\mathrm{v}}{dr} 2\pi r\right]_r$$

   The force on the outer boundary is

$$\left[\eta \frac{d\mathrm{v}}{dr} 2\pi r\right]_{r+dr} = \left[\eta \frac{d\mathrm{v}}{dr} 2\pi r\right]_r + dr\frac{d}{dr}\left(\eta \frac{d\mathrm{v}}{dr} 2\pi r\right).$$

   These forces are equal when

$$\frac{d}{dr}\left(\frac{d\mathrm{v}}{dr} r\right) = 0,$$

which implies $\mathrm{v} = C_1 \ln r + C_2$. Taking into account the boundary conditions $\mathrm{v}(R_1) = \mathrm{v}_0; \quad \mathrm{v}(R_2) = 0$, one gets finally

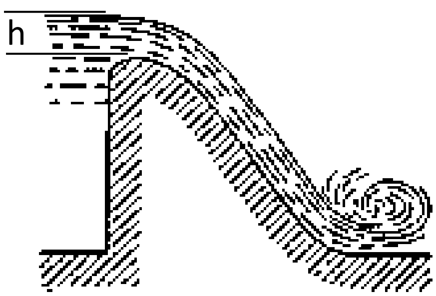
$$\mathrm{v} = \mathrm{v}_0 \frac{\ln \frac{R_2}{r}}{\ln \frac{R_2}{R_1}}.$$

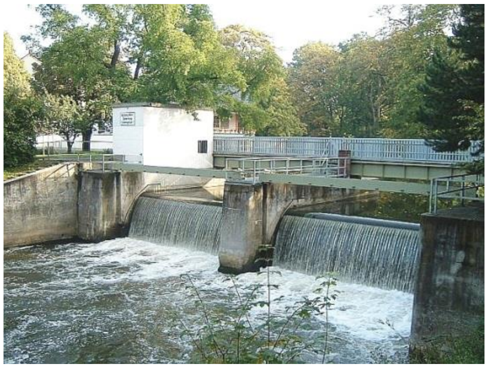
The force on the inner cylinder per unit length is

$$F = \left[\eta \frac{d\mathrm{v}}{dr} 2\pi r\right]_{r=R_1} = -\frac{2\pi\eta \mathrm{v}_0}{\ln \frac{R_2}{R_1}}$$

The force is decelerating.

2. **Water flows over a weir. Using dimensional analysis, find the dependence of the flow rate per unit width of the crest on the height of head of water over the crest, h. Note that because the Reynolds number of the flow is very large, the flow parameters are independent of viscosity.**



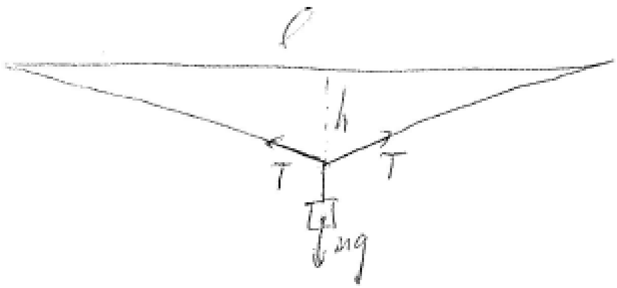


Dimensions of the flow rate per unit width of the crest: $[Q] = \left[\frac{V}{lt}\right] = \left[\frac{l^2}{t}\right]$

At *Re>>1,* the flow parameters are nearly independent of viscosity, therefore *Q* could depend only on *h* and *g; Q* could not depend on the density because *[Q]* does not contain mass. Comparing dimensions of *Q, h* and *g*, one gets

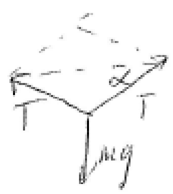
$$Q \propto \sqrt{gh^3}$$

3. **A wire of diameter *d* and length *l* is stretched horizontally. If a weight of mass *m* is suspended from the middle point of the wire, what is the resulting descent of the middle point , *h*? The proper weight of the wire could be neglected. Young's modulus of the wire is *Y.* Assume that *h<<l.***



$$\delta l = 2\sqrt{\left(\tfrac{l}{2}\right)^2 + h^2} - l = l\sqrt{1+\left(\tfrac{2h}{l}\right)^2} - l \approx \tfrac{2h^2}{l}$$

$$T = \pi\left(\tfrac{d}{2}\right)^2 Y\,\tfrac{\delta l}{l} = \tfrac{\pi}{2} d^2 Y\,\tfrac{h^2}{l^2}$$



Force balance: $mg = 2T\sin\alpha; \quad \sin\alpha = \dfrac{h}{\sqrt{\left(\tfrac{l}{2}\right)^2 + h^2}} \approx \dfrac{2h}{l}$

$$mg = 2\,\tfrac{\pi}{2}\,d^2 Y\,\tfrac{h^2}{l^2}\,\tfrac{2h}{l}$$

$$h = l\left(\frac{mg}{2\pi d^2 Y}\right)^{1/3}$$

4. **A spherical vessel with the radius *R* and the wall thickness *d<<R* is filled with a gas at the pressure *p>>p*$_{atm}$. Find the stress in the vessel?**

Let us consider the vessel as composed from two hemispherical envelopes. The excess gas pressure within the vessel pushes them apart whereas the elastic force between the hemispheres keeps them together. The last is expressed via the stress within the envelope as

$$F_{elast} = 2\pi R d\sigma\,.$$

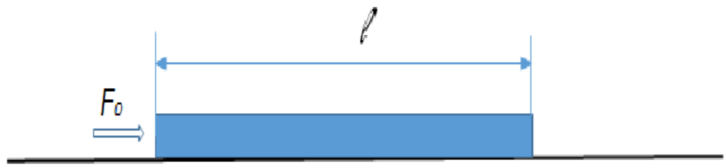
The pushing force due to the gas pressure may be found as (see question 2 from hw1)

$$F_p = \pi R^2 p\,.$$

Now the equilibrium condition, $F_{elast}=F_p$, yields

$$\sigma = \frac{Rp}{2d}\,.$$

5. מוט אחיד בעל אורך l ושטח חתך S עשוי מחומר עם מודול יאנג Y נעה על גבי שולחן בלי חיכוך בהשפעה של הכוח F0 הפועל על הצד של המוט כמתואר באיור. מצא\י את התפלגות המאמץ לאורך המוט וההתקצרות היחסית שלו, δl/l



The rod is accelerated, $a = F_0 / M$. At the distance x from the left end, the elastic force is developed such that the rest of the rod move with the same acceleration, $\sigma S = aM'$, where

$M' = \frac{l-x}{l} M$ is the mass of the rest of the rod. Now we find the stress as

$$\sigma = \frac{l-x}{l}\frac{F_0}{S}.$$

The corresponding deformation is $\xi = \sigma / Y$. Therefore the length of the rod decreases by

$$\delta l = \int_0^l \xi dx = \frac{l}{2}\frac{F_0}{YS}.$$

6. **Two identical steel rods collide with equal velocities *v* as it is shown in figure. Find the collision time if the length of rods is 10 cm. At what velocities plastic deformations arise? Density of steel is 7.8 g/cm^3, Young's modulus $2 \cdot 10^{11}$ Pa, the elastic limit $2 \cdot 10^{8}$ Pa.**

During the collision, the compressed region expands with the sound velocity

$$v_s = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{2 \cdot 10^{11}}{7.8 \cdot 10^{3}}} = 5 \cdot 10^{3} \text{ m/s}$$

The interaction time: $t = \frac{l}{\mathrm{v}_s} = \frac{0.1}{5 \cdot 10^3} = 2 \cdot 10^{-5}$ s. The rod is compressed by $\delta l = \mathrm{v}t = \frac{\mathrm{v}}{\mathrm{v}_s} l$. The stress in the rod is $\sigma = Y \frac{\delta l}{l} = Y \frac{\mathrm{v}}{\mathrm{v}_s}$. Plastic deformation arise at

$$\mathrm{v} = \frac{\sigma_{pl}}{Y} \mathrm{v}_s = \frac{2 \cdot 10^8}{2 \cdot 10^{11}} \cdot 5 \cdot 10^3 = 5 \text{ m/s}.$$