1. **While jogging, person's body produces energy at a rate of about 1500 W due to metabolism, 80% of which is converted to heat. For the runner to maintain a constant body temperature, this heat must be removed by perspiration. How much water must the person's body evaporate in an hour? The latent heat of vaporization for water at body temperature is 570 cal/g.**

$$\dot{q} = L\dot{m}$$

$$0.8 \cdot 1500 = 570 \cdot 4.2\dot{m}$$

$$\dot{m} = \frac{0.8 \cdot 1500}{4.2 \cdot 570} = 0.5 \text{ g/s=1.8 litr/h}$$

2. **At low temperatures, the specific heats of metals is described by the expression $c = \kappa T + AT^3$, where κ and *A* are constants. Here the first term describes the contribution of free electrons and the second the lattice contribution. How much heat is required to raise the temperature of 1 g of potassium from 1 K to 5 K? For potassium,** κ=2.1 mJ/(mol K^2), *A*=2.6 mJ/(mol K^4).

$$q = \frac{1}{\mu}\int_{T_1}^{T_2} CdT = \frac{1}{\mu}\int_{T_1}^{T_2}\left(\kappa T + AT^3\right)dT = \frac{1}{\mu}\left(\tfrac{1}{2}\kappa T^2 + \tfrac{1}{4}AT^4\right)_{T_1}^{T_2}$$

$$= \frac{1}{39}\left(\tfrac{1}{2}\cdot 2.1\cdot 10^{-3}\cdot 2.4 + \tfrac{1}{4}\cdot 2.6\cdot 10^{-3}\cdot 624\right) = 1.1\cdot 10^{-2} \text{ J}$$

3. **Julius Robert Mayer, who first enunciated the principle of the conservation of energy, estimated the mechanical equivalent of heat by making use of his relation between the heat capacities at constant volume and constant pressure. If for air, c_p=*0.241* cal/g K, c_V=0.172 cal/g K, what is the relation between the calorie and the joule?**

C_p-C_V=*R*; μ=*29*
C_p-C_V=*29 (0.241-0.172)=2.00* cal/mol K;   *R=8.31* J/mol K
1 cal = 4.15 J

4. **Two thermally insulated vessels with volumes V_1 and V_2, correspondingly, are filled with air. The temperature and pressure within the vessels are T_1, p_1 and T_2, p_2, correspondingly. What**

pressure and temperature are established after the vessels are connected? Hint: make use of the conservation of the total energy and mass.

Mass conservation: $\xi_1+\xi_2=\xi$

Energy conservation: $\xi_1 T_1+\xi_2 T_2=\xi T$

Making use of the equation of state, $pV=\xi RT$, one gets

$$\begin{cases} \dfrac{p_1V_1}{T_1}+\dfrac{p_2V_2}{T_2}=\dfrac{p(V_1+V_2)}{T} \\ p_1V_1+p_2V_2=p(V_1+V_2) \end{cases}$$

This yields $p=\dfrac{p_1V_1+p_2V_2}{V_1+V_2}; \quad T=\dfrac{p_1V_1+p_2V_2}{\frac{p_1V_1}{T_1}+\frac{p_2V_2}{T_2}}$

5. **A vertical cylinder of radius *r* contains a quantity of ideal gas and is fitted with a piston with mass *m* that is free to move without friction. The cylinder is open, above the piston, to the air. In equilibrium, the piston sits at a height *h* above the bottom of the cylinder. (a) Find the pressure of the gas below the piston when in equilibrium. (b) The piston is pulled up by a small distance *x*<<*h*. Find the net force acting on the piston assuming that there is no heat exchange between the gas and surroundings. (c) After the piston is displaced from equilibrium and released, it oscillates up and down. Find the frequency of these small oscillations neglecting the mass of the gas in the cylinder with respect to the mass of the piston.**

a) $$p_0=p_{\text{atm}}+\frac{mg}{\pi r^2}$$

b) $pV^{\gamma}=p_0V_o^{\gamma}; \quad V_0=\pi r^2 h$ adiabatic process

$$p=p_0\left(\frac{V_0}{V}\right)^{\gamma}$$

$$\delta p=\left.\frac{dp}{dV}\right|_{V=V_0}\delta V=-\gamma p_0\frac{\delta V}{V_0}=-\gamma p_0\frac{x}{h}$$

$$F=\pi r^2\delta p=-\pi\gamma r^2 p_0\frac{x}{h}$$

c) *ma=F*

$$m\frac{d^2x}{dt^2}+\pi\gamma r^2\frac{p_0}{h}x=0$$

$$\omega^2=\frac{\pi\gamma r^2 p_0}{mh}$$

$$f=\frac{\omega}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{\gamma g}{h}\left(1+\frac{\pi r^2 p_{\mathrm{atm}}}{mg}\right)}$$