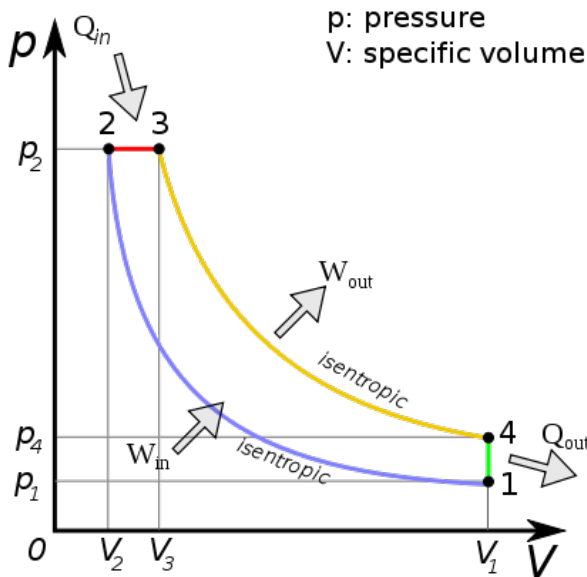


1. Consider the idealized Diesel cycle (see figure). Starting at point 1, air is compressed adiabatically to point 2. At the end of the compression step (point 2), the injectors start to inject fuel into the cylinder. Because of the high temperature developed during the adiabatic compression, the fuel ignites spontaneously as it is injected. The piston is pushed by the hot gas so that the gas expands under the constant pressure till point 3, after which it expands adiabatically. The gas is cooled to the temperature of the outside air along the line 4-1. Derive an expression for the efficiency of the cycle in terms of the compression ratios  $w=V_3/V_1$  and  $r=V_2/V_1$  and the ratio of specific heats,  $\gamma$ .



$$Q = \int (dE + p dV); \quad pV = \xi RT; \quad E = \xi C_V T = \frac{C_V}{R} pV$$

$$Q_{in} = \int \left( \frac{C_V}{R} + 1 \right) p dV = \frac{C_V + R}{R} p_2 (V_3 - V_2)$$

$$= \frac{C_p p_2 V_1}{R} (w - r); \text{ here I used Mayer's equation, } C_p - C_V = R$$

$$Q_{out} = E_4 - E_1 = \frac{C_V}{R} V_1 (p_4 - p_1) = \frac{C_V}{R} p_2 V_1 (w^\gamma - r^\gamma) \text{ here I used}$$

Poisson's adiabata,  $pV^\gamma = const$

$$e = \frac{W}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{1}{\gamma} \frac{w^\gamma - r^\gamma}{w - r}$$

2. A chunk of ice of mass 50 g at a temperature 0°C is added to 250 g of water at a temperature 30°C. What is the entropy increment in the system by the moment when the thermal equilibrium is established? The heat of fusion of ice is 80 cal/g.

$$m=50 \text{ g}; M=250 \text{ g}; t_1=0^\circ\text{C}; t_2=30^\circ\text{C}; q=80 \text{ cal/g}; c=1 \text{ cal/g}$$

Let the final temperature be  $t$ . The heat balance,

$$cM(t_2 - t) = mq + mc(t - t_1), \text{ yields}$$

$$t = \frac{Mt_2 - mq/c}{M + m} = \frac{250 \cdot 30 - 80 \cdot 50}{300} \approx 11.7^\circ\text{C}$$

$$\delta S_1 = \frac{qm}{T_1} = \frac{80 \cdot 50}{273} \approx 14.7 \text{ cal/grad}$$

$$\delta S_2 = \int \frac{dQ}{T} = cm \int_{T_1}^T \frac{dT}{T} = cm \ln \frac{T}{T_1} = 50 \cdot \ln \frac{273 + 11.7}{273} = 2.1 \text{ cal/grad}$$

$$\delta S_3 = cM \ln \frac{T}{T_2} = 250 \cdot \ln \frac{273 + 11.7}{273 + 30} = -15.6$$

$$\delta S = 14.7 + 2.1 - 15.6 = 1.2 \text{ cal/grad}$$

3. Real gases are described by the Van der Waals equation of state; for a mole it is written as

$$\left( p + \frac{a}{V^2} \right) (V - b) = RT$$

where  $a$  and  $b$  are constants. At large volumes (i.e. at small gas densities), this equation is reduced to that for ideal gas. Making use of the thermodynamic relation

$$\left( \frac{\partial E}{\partial V} \right)_T = T \left( \frac{\partial p}{\partial T} \right)_V - p$$

find the internal energy of the Van der Waals gas. Hint: take into account that at small densities, the internal energy is that of ideal gas,  $E = C_V T$ .

$$p = \frac{RT}{V-b} - \frac{a}{V^2}$$

$$\left(\frac{\partial p}{\partial T}\right)_V = \frac{R}{V-b};$$

Now the thermodynamic identity yields  $\left(\frac{\partial E}{\partial V}\right)_T = \frac{a}{V^2}$

$$E = \int \frac{adV}{V^2} = -\frac{a}{V} + f(T)$$

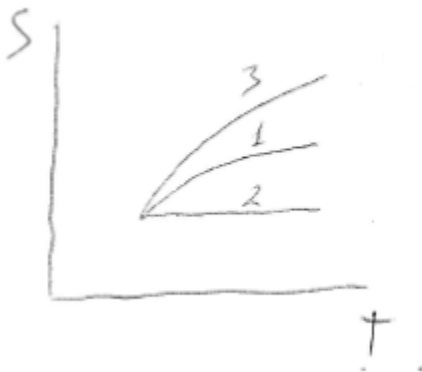
$E = C_v T$  when  $V \rightarrow \infty$  therefore finally  $E = C_v T - \frac{a}{V}$

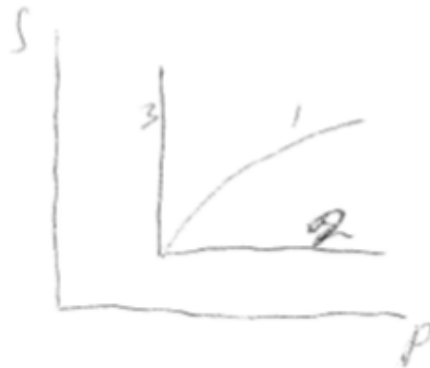
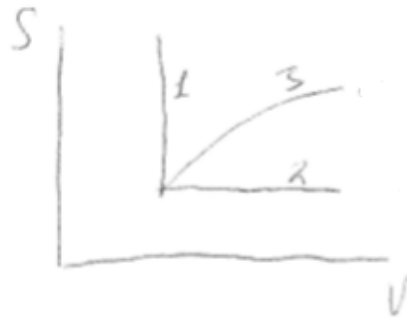
4. Plot approximate graphs of isochoric, adiabatic and isobaric processes for an ideal gas in coordinate systems a) (S, T); b) (S, V); c) (S, p).

1. Isochoric:  $\delta S = \int \frac{dQ}{T} = C_v \int \frac{dT}{T} = C_v \ln T$

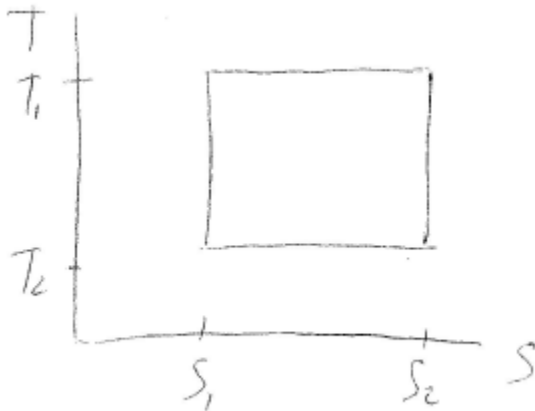
2. Adiabatic:  $\delta S = 0$

3. Isobaric:  $\delta S = C_p \int \frac{dT}{T} = C_p \ln T$





5. Plot graph of the Carnot cycle in the coordinate system (T, S). Find the efficiency of the cycle from this graph.



$$dQ = TdS$$

$$Q_{in} = T_1(S_2 - S_1); \quad Q_{out} = T_2(S_2 - S_1)$$

$$e = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1}$$