

CLASS EX. 1: FOURIER RECAP

We define the fourier of a L-preiodic function $f(x) = f(x + L)$ by:

$$f(x) = \sum_{k_n} f(k) e^{ikx}$$

$$f(k_n) = \frac{1}{L} \int_{-L/2}^{L/2} f(x) e^{-ikx} dx$$

$$k = (2\pi/L)n \quad n = \dots, -1, 0, 1, \dots$$

1.

- a) For $g(x) = e^{ik_0 x} f(x)$, write $g(k)$ in terms of $f(k)$ (What are the constraint on k_0 ?)
 b) For $f(x) = \delta(x - x_0)$. Find $f(k)$. Write $f(x)$ as a sum over k (this is a series representation for the delta function).

2.

- a) Find the Fourier transform and it's inverse in the limit $L \rightarrow \infty$ in Eq.(1). Redefine $f(k)$ if necessary. How does the $\delta(x - x_0)$ from the last question looks like in this limit.
 b) Write the Fourier transform of N delta function spaced distance a from each other. Check the limit of $N \rightarrow \infty$.

3.

What is the fourier transform of $f(x) = \exp(-(x/\sigma)^2)$, $x \in [-L/2, L/2]$. Assume $L/\sigma \gg 1$.

4.

Draw the function $f(x) = 1 - \frac{2}{L}|x|$, for $x \in [-L/2, L/2]$. Find it's fourier transform.