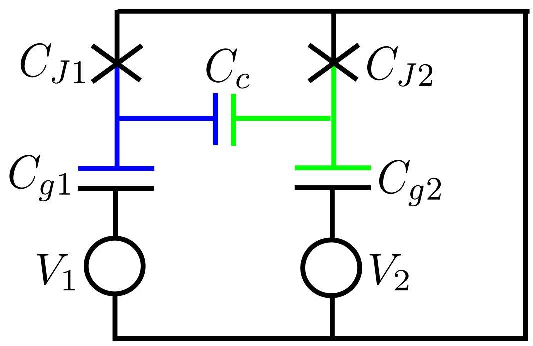
Selected topics in solid state physics 2
Exercise 2

I. TWO COUPLED CHARGE QUBITS



In the picture above you can see a possible realization of two charge qubits, which are coupled to one another via a capacitance C_c, the island of the first qubit is colored blue and that of the second qubit is colored green.

(a): As in the lecture, assign a phase to each element (in units of magnetic flux) and write down the Lagrangian. Take into account that the Josephson junctions, are characterized by Josephson energies E_{J1} and E_{J2} (not indicated in the picture) and by capacitances C_{J1} and C_{J2} respectively. Eliminate some variables using Kirchhoff's rules. How many independent degrees of freedom remain? Carry out a Legendre transformation and determine the Hamiltonian of the system.

(b): Perform the canonical quantization. Show, that in the regime of dominant charging energy ($E_C \gg E_J$) only two charge states are relevant is each island (for certain choice of gate voltages V_1 and V_2). Determine the 4-dimensional Hamiltonian of the two coupled charge qubits.

II. NUMERICAL DIAGONALIZATION OF THE CHARGE QUBIT HAMILTONIAN

In the lecture we have obtained the following Hamiltonian of the charge qubit

$$H = E_C(n - q_g)^2 - E_J \cos\phi \ . \tag{1}$$

Here n is the number of extra Cooper pairs on the island, q_g is the dimensionless gate charge ($q_g = Q_g/2e$). The operator $e^{i\phi}$ is defined by $e^{i\phi}|n\rangle = |n+1\rangle$. Perform numerical diagonalization of the above Hamiltonian (you can use Mathematica for example). Use the charge basis $|n\rangle$ and restrict the number of basis states (using physical arguments) in order to obtain a matrix of a finite dimension. Plot 3 lowest eigen-energies as functions of q_g. Investigate the regimes: a) $E_J \ll E_C$, b) $E_J \sim E_C$, c) $E_J \gg E_C$.