Selected topics in solid state physics 2
Exercise 3

I. RELATION BETWEEN T_1 AND T_2 TIMES

Consider the Lindblad equation derived in the class. Consider the general case of a qubit (two-level system) coupled longitudinally and transversely to two uncorrelated noise sources (baths). The Hamiltonian then reads

$$H = -\frac{1}{2}\,\Delta E\sigma_z - \frac{1}{2}\,X\sigma_x - \frac{1}{2}\,Y\sigma_z + H_{Bath,X} + H_{Bath,Y}\ .$$

The noise variables X and Y are uncorrelated. Assume for definiteness that both baths consist of continuum of oscillators and are characterized by the Ohmic spectral functions $J_X(\omega) = \alpha_X\omega$, $J_Y(\omega) = \alpha_y\omega$ (see the definition of the spectral function in the part of the lecture notes devoted to the Caldeira-Leggett model). Write down the appropriate Linblad equation, including the Lamb shift. Calculate the relaxation rates T_1^{-1} and T_2^{-1} as well as the Lamb shift (assume a frequency cut-off if needed).

II. T_1 AND T_2 TIMES FOR A DRIVEN TWO-LEVEL SYSTEM

Consider now a driven qubit:

$$H = -\frac{1}{2}\,\Delta E\sigma_z - \Omega\cos(\omega t)\sigma_x - \frac{1}{2}\,X\sigma_x - \frac{1}{2}\,Y\sigma_z + H_{Bath,X} + H_{Bath,Y}\ .$$

The most interesting case is that of resonance, when $\hbar\omega$ is close to ΔE. Assume this is the case but allow for detuning, i.e., $\delta \equiv \Delta E - \hbar\omega \neq 0$. Perform the transition to the rotating frame (the appropriate frame in the one rotating with angular frequency ω around the z-axis), make the non-dissipative part of the Hamiltonian time-independent (use RWA) and find the relaxation rates T_1^{-1} and T_2^{-1} in the rotating frame.