

Formula sheets

Introduction to Mathematical Methods in Physics (203.1.1141)
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Trigonometric identities

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

$$\sin \alpha \pm \sin \beta = 2 \sin \frac{\alpha \pm \beta}{2} \cos \frac{\alpha \mp \beta}{2}$$

$$\sin \alpha \sin \beta = \frac{1}{2} (\cos(\alpha - \beta) - \cos(\alpha + \beta))$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\cos \alpha \cos \beta = \frac{1}{2} (\cos(\alpha - \beta) + \cos(\alpha + \beta))$$

$$\cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

$$\sin \alpha \cos \beta = \frac{1}{2} (\sin(\alpha - \beta) + \sin(\alpha + \beta))$$

$$\tan \alpha \pm \tan \beta = \frac{\sin(\alpha \pm \beta)}{\cos \alpha \cos \beta}$$

$$\tan^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{1 + \cos \alpha}$$

$$\sin^2 \frac{\alpha}{2} = \frac{1}{2} (1 - \cos \alpha)$$

$$\cos^2 \frac{\alpha}{2} = \frac{1}{2} (1 + \cos \alpha)$$

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

Hyperbolic functions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\cosh^2 x - \sinh^2 x = 1$$

Derivatives

$$(x^n)' = nx^{n-1}$$

$$(\ln x)' = \frac{1}{x}$$

$$(e^x)' = e^x$$

$$(a^x)' = a^x \ln a$$

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

$$(\tan x)' = \frac{1}{\cos^2 x}$$

$$(\cot x)' = -\frac{1}{\sin^2 x}$$

$$(\arcsin x)' = -(\arccos x)' = \frac{1}{\sqrt{1-x^2}}$$

$$(\arctan x)' = -(\operatorname{arccot} x)' = \frac{1}{1+x^2}$$

$$(\sinh x)' = \cosh x$$

$$(\cosh x)' = \sinh x$$

$$(\tanh x)' = \frac{1}{\cosh^2 x}$$

$$(\operatorname{coth} x)' = -\frac{1}{\sinh^2 x}$$

Integrals

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \frac{x - a}{x + a} \qquad \int \frac{dx}{x^2 + a^2} = \frac{1}{a} \arctan \frac{x}{a}$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left(x + \sqrt{x^2 \pm a^2} \right) \qquad \int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{|a|}$$

$$\int \tan x \, dx = -\ln \cos x \qquad \int \cot x \, dx = \ln \sin x$$

$$\int \tanh x \, dx = \ln \cosh x \qquad \int \coth x \, dx = \ln \sinh x$$

$$\int \frac{dx}{\sin x} = \ln \tan \frac{x}{2} \qquad \int \frac{dx}{\cos x} = \ln \left(\tan x + \frac{1}{\cos x} \right)$$

Taylor series

$$(1 + x)^p = 1 + px + \frac{p(p-1)}{2!} x^2 + \dots + \frac{p(p-1)(p-2)\dots(p-n+1)}{n!} x^n + \dots$$

$$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \dots + (-1)^{n+1} \frac{x^n}{n} + \dots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + (-1)^n \frac{x^{2n}}{(2n)!} + \dots$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + \frac{x^{2n+1}}{(2n+1)!} + \dots$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + \frac{x^{2n}}{(2n)!} + \dots$$

Vectors

$$\delta_{ij} = \delta_{ji} \qquad \epsilon_{ijk} = -\epsilon_{jik} = -\epsilon_{ikj} = -\epsilon_{kji} \qquad \epsilon_{ijk} = \epsilon_{kij} = \epsilon_{jki} \qquad \epsilon_{ijk}\epsilon_{ilm} = \delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}$$

$$\vec{A} \cdot \vec{B} = A_i B_i = \delta_{ij} A_i B_j \qquad (\vec{A} \times \vec{B})_i = \epsilon_{ijk} A_j B_k$$

Gauss' and Stokes' theorems

$$\iiint_V \vec{\nabla} \cdot \vec{F} \, dV = \iint_S \vec{F} \cdot \hat{n} \, dS \qquad \iint_S (\vec{\nabla} \times \vec{F}) \cdot \hat{n} \, dS = \oint_C \vec{F} \cdot d\vec{r}$$

Coordinate systems

polar

$$x = r \cos \phi$$

$$y = r \sin \phi$$

$$dA = r dr d\phi$$

cylindrical

$$x = r \cos \phi$$

$$y = r \sin \phi$$

$$z = z$$

$$dV = r dr d\phi dz$$

spherical

$$x = r \sin \theta \cos \phi$$

$$y = r \sin \theta \sin \phi$$

$$z = r \cos \theta$$

$$dV = r^2 \sin \theta dr d\theta d\phi$$

vector operators in cylindrical coordinates

$$\vec{\nabla} f(r, \phi, z) = \frac{\partial f}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial f}{\partial \phi} \hat{\phi} + \frac{\partial f}{\partial z} \hat{z}$$

$$\vec{\nabla} \cdot \vec{A}(r, \phi, z) = \frac{1}{r} \frac{\partial}{\partial r} (r A_r) + \frac{1}{r} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\vec{\nabla} \times \vec{A}(r, \phi, z) = \left(\frac{1}{r} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) \hat{r} + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) \hat{\phi} + \frac{1}{r} \left[\frac{\partial (r A_\phi)}{\partial r} - \frac{\partial A_r}{\partial \phi} \right] \hat{z}$$

vector operators in spherical coordinates

$$\vec{\nabla} f(r, \theta, \phi) = \frac{\partial f}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial f}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi} \hat{\phi}$$

$$\vec{\nabla} \cdot \vec{A}(r, \theta, \phi) = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial (A_\theta \sin \theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi}$$

$$\vec{\nabla} \times \vec{A}(r, \theta, \phi) = \frac{1}{r \sin \theta} \left[\frac{\partial (A_\phi \sin \theta)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi} \right] \hat{r} + \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial A_r}{\partial \phi} - \frac{\partial (r A_\phi)}{\partial r} \right] \hat{\theta} + \frac{1}{r} \left[\frac{\partial (r A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta} \right] \hat{\phi}$$

Jacobian

$$dx dy \rightarrow \left\| \begin{array}{cc} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{array} \right\| du dv$$

Area element on surface $z = f(x, y)$: $dS = \sqrt{1 + \left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2} dx dy$

Rotation matrices

$$R = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

$$R_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{pmatrix} \quad R_y = \begin{pmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{pmatrix} \quad R_z = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$