QFT Home Assignment # 2. Submission date 12.12.2021

1. Examine $\langle 0|U|0\rangle$ to order λ^2 in ϕ^4 theory. Identify the different diagrams with the different contributions arising from an application of Wick's theorem. Confirm that to order λ^2, the combinatoric factors work out so that the vacuum to vacuum amplitude is given by the exponential of the sum of distinct vacuum bubbles.

2. Consider Lagrangian for 3 scalar fields ϕ_i, $i = 1, 2, 3$, given by

$$L = \sum_{i=1}^{3} \frac{1}{2}(\partial_\mu \phi_i)(\partial^\mu \phi_i) - \frac{1}{2} m^2 \sum_{i=1}^{3} (\phi_i)^2 - \frac{\lambda}{8} \left(\sum_{i=1}^{3} (\phi_i)^2 \right)^2 \tag{1}$$

Write down the Feynman rules of the theory. Compute the amplitude for the scattering $\phi_i \phi_j \to \phi_k \phi_l$ to lowest non-trivial order in λ.

3. Consider a real scalar field theory with the Lagrangian density

$$L = \frac{1}{2} (\partial_\mu \phi)^2 - \frac{\lambda}{4} (\phi^2 - v^2)^2 \tag{2}$$

where λ and v are some positive real numbers. Imagine that the theory lives in 1 + 1 space-time dimensions labeled (t, x). Construct the Hamiltonian for the theory. Show that, for time-independent fields $\phi(t, x) = \phi(x)$, the energy minima (the vacua) of the Hamiltonian are given by

$$\phi = \pm v$$

Find the time-independent solution of the equations of motion that interpolates between the two vacua. That is find the solution $\phi(x)$ satisfying the following conditions

$$\phi(x = -\infty) = -v; \quad \phi(x = +\infty) = v.$$

(You may also require that $\phi(x = 0) = 0$ for simplicity.) Such solution is known as the kink solution and is the simplest example of a soliton.

4. Consider the Lagrangian density

$$L = \frac{1}{2} (\partial_\mu \phi)^2 - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{4!} \phi^4 \tag{3}$$

Draw all possible connected Feynman diagrams up to the order λ^3 contributing to (a) the two-point correlator, (b) the three-point correlator, (c) the four-point correlator. Find the symmetry factors for all the graphs. Restore $\hbar$ and find the relation between $\hbar$ and loop expansions.

5. Consider the Lagrangian density

$$L = \frac{1}{2} (\partial_\mu \phi)^2 - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{3!} \phi^3 \tag{4}$$

Draw all possible connected Feynman diagrams contributing to (a) the two-point correlator (up to the order λ^4), (b) the three-point correlator (up to the order λ^3). Find the symmetry factors for all the graphs. Restore $\hbar$ and find the relation between $\hbar$ and loop expansions.