QFT Home Assignment # 4. Submission date 09.01.2022

0. Fill gaps through self-education: read Peskin and Schroeder, Chapter 11, 12.4, 12.5.

1. Peskin and Schroeder: Problem 10.3.

2. Peskin and Schroeder: Problem 10.4.

3. Consider ϕ^4 scalar theory with the Lagrangian

$$L = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{4!}\phi^4$$

Below use Pauli-Villars regularization with two new particles, such that the propagators of the internal lines are replaced by

$$\frac{i}{k^2 - m^2 + i\epsilon} \to \frac{i}{k^2 - m^2 + i\epsilon} + \sum_{i=1}^{2} \frac{iC_i}{k^2 - M_i^2 + i\epsilon}$$

with $C_1 = 1$, $C_2 = -2$, and $M_1^2 = m^2 + 2M^2$, $M_2^2 = m^2 + M^2$. M is some UV cutoff.

a. Calculate regularized one-loop corrections to the propagator and to the connected four-point Green function, keeping only divergent and infinite terms at large M.

b. Set up the renormalization program for the theory at one loop. Find all counterterms and coefficients in front of them using "on-shell" renormalization conditions. Find the renormalized truncated 4-point Green function $\Gamma^4(s;t;u)$ up to order λ^2. Choose the counter-terms such that $\Gamma^4(s,t,u) = -i\lambda$ when $s = t = u = 4m^2$