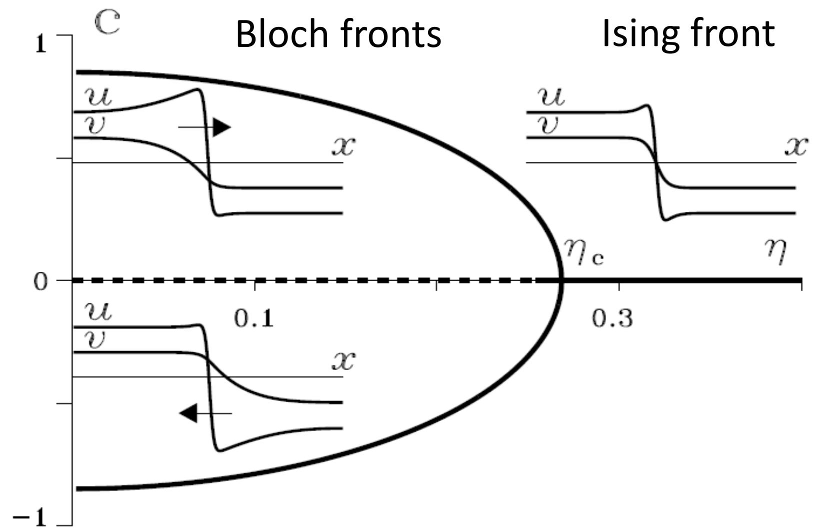
Longitudinal front instability (nonequilibrium Ising-Bloch bifurcation):

$$c = \frac{3c}{\sqrt{2q^2}\sqrt{c^2+4\eta^2 q^2}} + c_\infty,$$

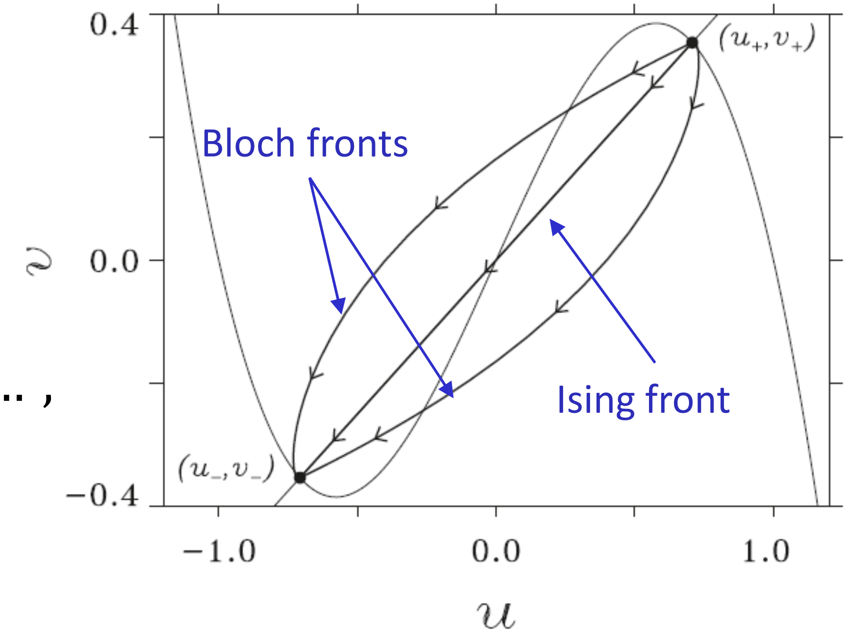
$$c_\infty = -\frac{3a_0}{\sqrt{2q^2}}, \qquad q^2 = a_1 + 1/2$$



For the symmetric FHN model ($a_0 = 0 \rightarrow c_\infty = 0$):

$$c = 0, \qquad c = \pm 2q\sqrt{\eta_c^2 - \eta^2}$$

$$, \qquad \eta_c^2 = \frac{9}{8q^3}$$



$$u(\chi) = u_0 + cu_1 + \ \dots =$$

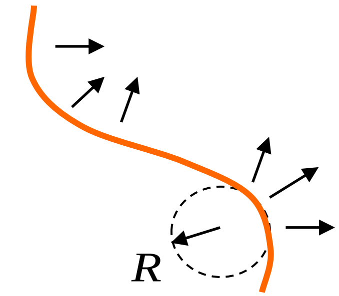
$$-\tanh\left(\frac{\zeta}{\sqrt{2\mu}}\right) + \frac{1}{3}c\cdot\mathrm{sech}^2\left(\frac{\zeta}{\sqrt{2\mu}}\right) + \ \dots ,$$

$$\zeta = \sqrt{\mu}\chi = \sqrt{\mu}(x - ct)$$

Transverse front instability: Front curvature affects the diffusion of the activator and the inhibitor and thereby the front velocity c:

$$c = c(\kappa) \qquad \kappa = 1/R$$

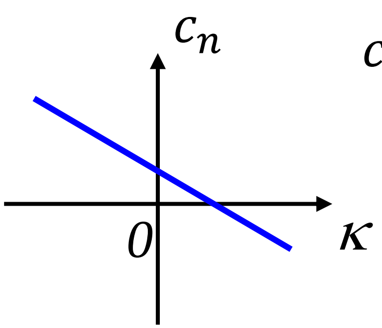
← Radius of curvature



We found:

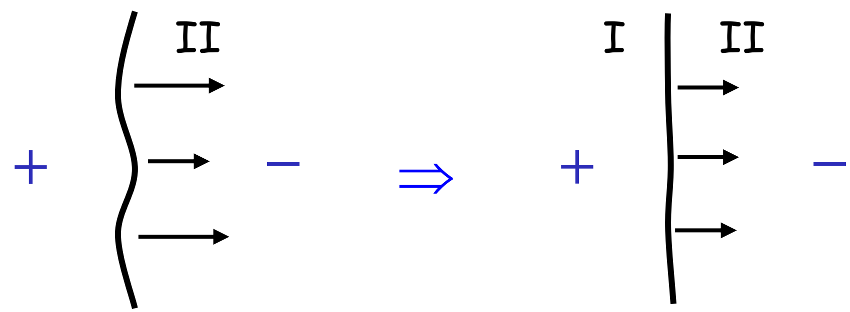
$$c_n + \kappa = \frac{3(c_n + \delta\kappa)}{\sqrt{2}q^2\sqrt{(c_n + \delta\kappa)^2 + 4\eta^2 q^2}} + c_\infty \qquad \delta = D_v/D_u$$

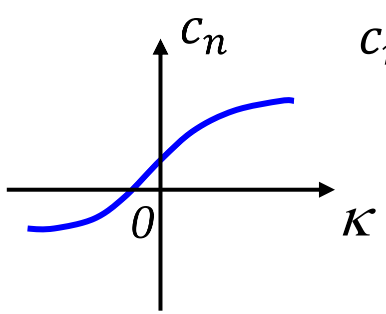
For $|\kappa| \ll 1$:



$$c_n = c_0 - D\kappa + \cdots$$

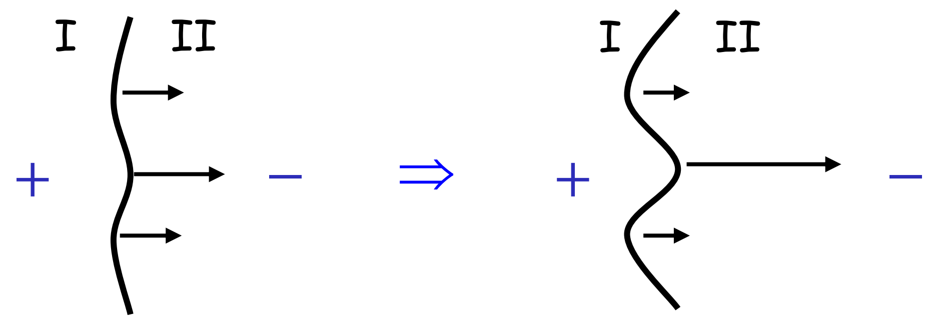
When $D > 0$
(δ small)
Front is stable





$$c_n = c_0 + |D|\kappa + \ \ldots$$

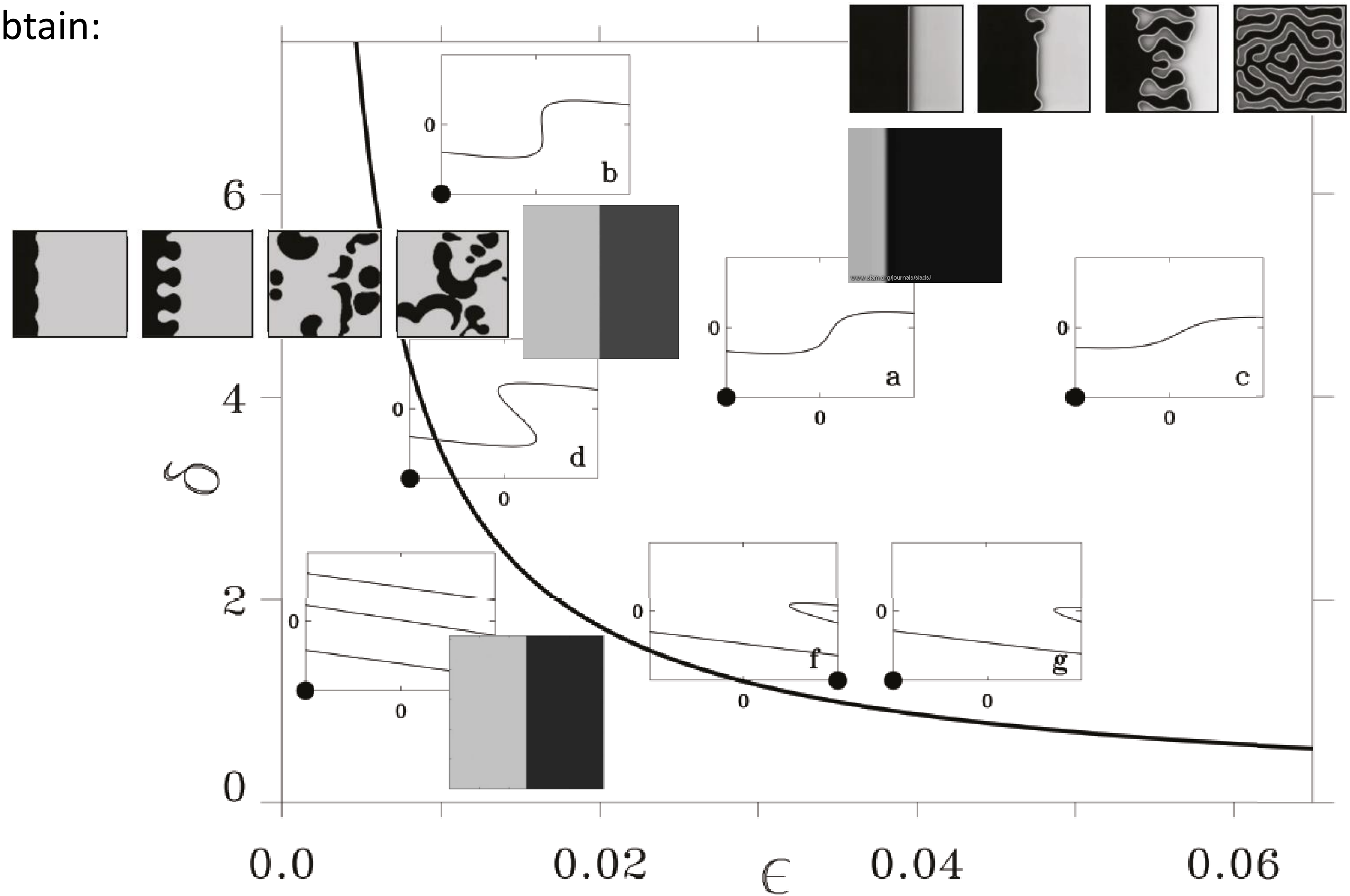
When $D < 0$
(δ large)
Front is unstable



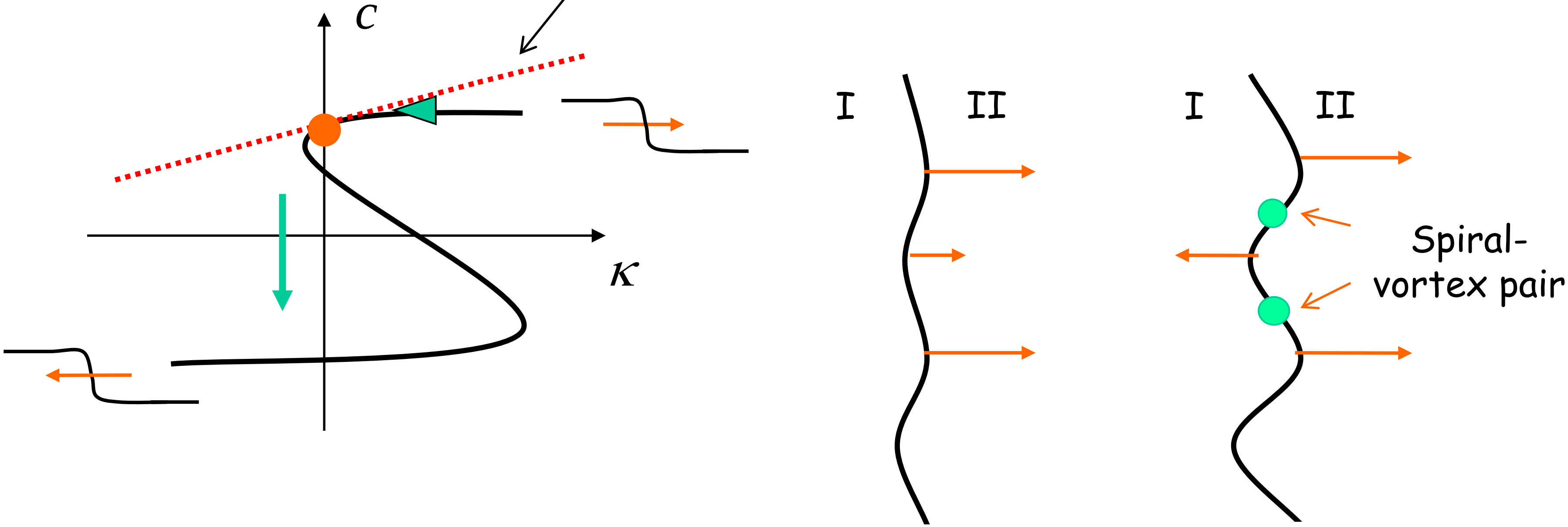
Use

$$c_n + \kappa = \frac{3(c_n + \delta\kappa)}{\sqrt{2}q^2\sqrt{(c_n + \delta\kappa)^2 + 4\eta^2 q^2}} + c_\infty$$

to obtain:



Positive slope at $\kappa = 0$ implies transverse instability



Growing transverse perturbations

Local front transitions & spiral vortex nucleation