# Fine structure:

- So far we discussed only the gross structure. Considering only kinetic energy, electron nuclear attraction and electron – electron repulsion.
- Smaller interaction – due to magnetic effects. At this stage – only internal magnetic fields – resulting in fine structure.

Fine structure – magnetic interaction between the orbital and spin magnetic moments

Hyperfine structure – magnetic interaction between electron and nuclear magnetic moments.

# The quantum mechanical solution

- The magnitude of the orbital angular momentum is: $L = \sqrt{l(l+1)}\hbar$, l takes positive integer values up to n-1
- Component along a certain axis (normally z):

$$L_z = m_l \hbar$$

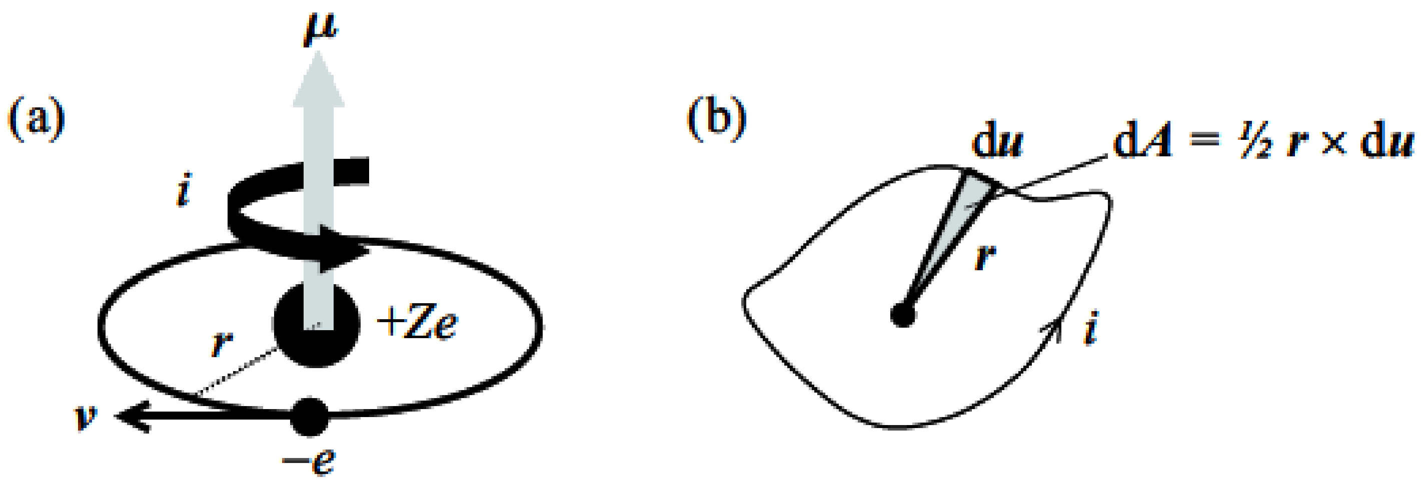
- m takes integer values from –l to l.
- As discussed already L creates a magnetic dipole μ.

$$\mu = i \times \text{Area} = -(e/T) \times (\pi r^2)$$

$$\mu = -\frac{ev}{2\pi r}\pi r^2 = -\frac{e}{2m_e}m_e v r = -\frac{e}{2m_e}L,$$

# Generalization of the expression for the gyromagnetic ratio:

- We know that the orbit is not always circular:




- The magnetic dipole is given by the integral:

$$\mu = \oint i \, \mathrm{d}A$$

- The incremental area is: $\mathrm{d}A = \frac{1}{2} r \times \mathrm{d}u$ and

$$\mu = \frac{1}{2} \oint i \, r \times \mathrm{d}u$$

# Continued:

- We write I = -e/T

and get: $$= \frac{1}{2}\oint \mathrm{d}q\, \boldsymbol{r} \times \frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t} = \frac{1}{2}\oint \mathrm{d}q\, \boldsymbol{r} \times \boldsymbol{v}\,,$$

$$= \frac{1}{2m_\mathrm{e}}\oint \mathrm{d}q\, \boldsymbol{r} \times \boldsymbol{p}\,,$$

Recall: $$\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p}$$

We get: $$\boldsymbol{\mu} = \frac{1}{2m_\mathrm{e}}\oint \boldsymbol{L}\,\mathrm{d}q = \frac{1}{2m_\mathrm{e}}\boldsymbol{L}\oint \mathrm{d}q = \frac{1}{2m_\mathrm{e}}\boldsymbol{L}(-e)$$

$$|\boldsymbol{\mu}| = \frac{e}{2m_\mathrm{e}}\hbar\sqrt{l(l+1)} = \mu_\mathrm{B}\sqrt{l(l+1)}\,,$$

(This is possible because L is a constant of the motion). $e/2m_e$ – The gyromagnetic ratio.

The magnitude of μ: $$|\boldsymbol{\mu}| = \frac{e}{2m_\mathrm{e}}\hbar\sqrt{l(l+1)} = \mu_\mathrm{B}\sqrt{l(l+1)}\,,$$

# The Bohr magneton:

- The unit of magnetic dipole:

$$\mu_{\rm B} = \frac{e\hbar}{2m_{\rm e}} = 9.27 \times 10^{-24}\,{\rm JT}^{-1}.$$

Mostly we are interested in the z component:

$$\mu_{\rm z} = -\frac{e}{2m_{\rm e}} L_{\rm z} = -\mu_{\rm B} m_l$$

$m_l$ the magnetic quantum number.

Spin magnetism: The deflection from the Stern Gerlach experiment:

$$\mu_{\rm z} = -g_{\rm s} \mu_{\rm B} m_s$$

# $g_e$ the g value of the electron:

- $m_s$ =±1/2 is the magnetic quantum numbe of the spin. $g_e$ is predicted by the Dirac eqn. to be 2.
- Quantum elecrodynamics gives $2.0023192$, in agreement with experiments.
- 

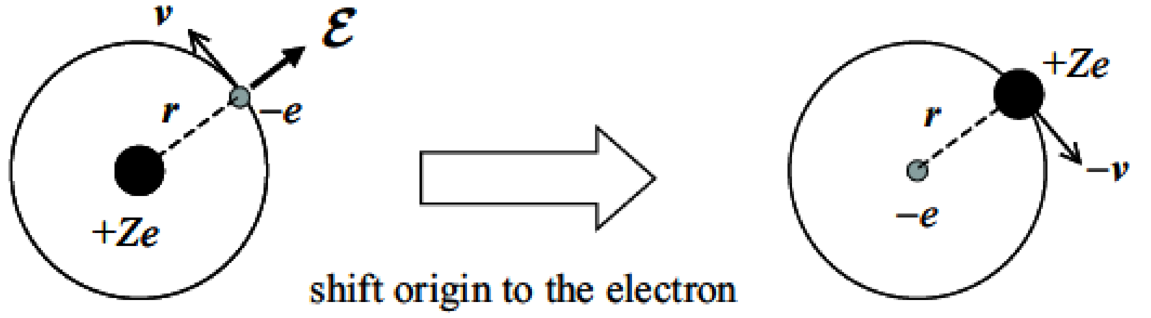
# Spin orbit coupling:

- Actually – a relativistic effect (like the spin) But a more intuitive explanation –

  the interaction between the magnetic field by the orbit and the magnetic moment of the spin.

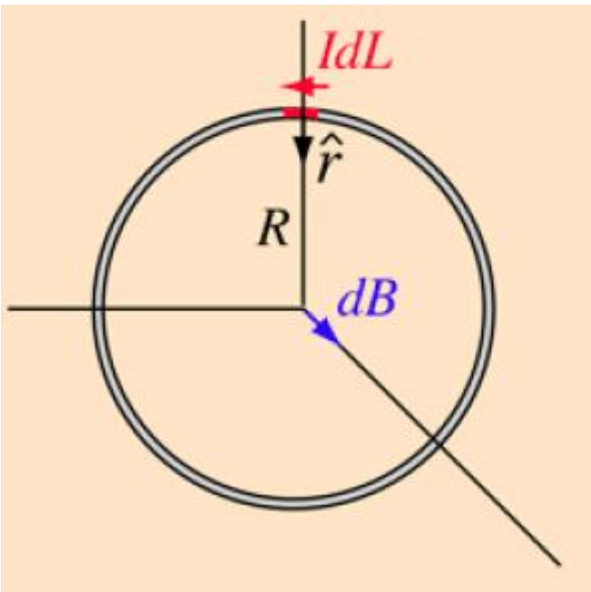
  The coupling can be estimated using the Bohr model. A better result – with modern quantum mechanics.

  We assume a static electron and a nucleus rotating around it.



# Spin-orbit coupling in the Bohr model

Current loop calculated by Biot Savart law




$$dB = \frac{\mu_0 I d\vec{L} \times \vec{\hat{r}}}{4\pi R^2} = \frac{\mu_0 I dL \sin\theta}{4\pi R^2}$$

$$B = \frac{\mu_0 I}{4\pi R^2} \oint dL = \frac{\mu_0 I}{4\pi R^2} 2\pi R = \frac{\mu_0 I}{2R}$$

$$\mu_0 = 4\pi \ \ x \ \ 10^{-7} T \cdot m / A$$

In the direction perpendicular to the loop (z)

$$B_z = \frac{\mu_0 i}{2r}$$

# Cont.

- I given by charge Ze divided by orbital period:

$$T = 2\pi r/v.$$

We get. $$B_z = \frac{\mu_0 Z e v_n}{4\pi r_n^2}$$

Recall: according to the Bohr model:

$$r_n = \frac{n^2}{Z}\frac{m_e}{m}a_0 \qquad v_n = \alpha\frac{Z}{n}c.$$

- Where the Bohr radius and the fine structure constant are:

$$a_0 = \frac{h^2\epsilon_0}{\pi m_e e^2} \qquad \alpha = \frac{e^2}{2\epsilon_0 hc}$$

# The field induced by the nucleus

$$B_z = \frac{\mu_0 Z e v_n}{4\pi r_n^2} = \left(\frac{Z^4}{n^5}\right)\frac{\mu_0 \alpha c e}{4\pi a_0^2}$$

- Where: $\alpha = e^2/2\epsilon_0 hc \approx 1/137$
- For hydrogen with Z=n=1, $B_z$ =12 T.
- The interaction of the electron with this field:

$$\Delta E_{so} = -\mu_{spin} \cdot B_{orbital} \,;$$

- Calculating the z component:

$$\Delta E_{so} = g_s \mu_B m_s B_z = \pm \mu_B B_z$$

- (Using $g_s = 2$ and $m_s = \pm 1/2$ ).

# Using

- The equations

$$\mu_{\rm B} = \frac{e\hbar}{2m_{\rm e}} = 9.27\times10^{-24}\,{\rm JT}^{-1}.$$

- We get $|\Delta E_{\rm so}| = \left(\frac{Z^4}{n^5}\right)\frac{\mu_0\alpha c e^2\hbar}{8\pi m_{\rm e}a_0^2} \equiv \alpha^2\frac{Z^2}{n^3}|E_n|$;

- Where we used:

- $E_n = -\frac{mZ^2e^4}{8\epsilon_0^2h^2n^2}$

# The energy is expressed in terms of Rydberg energy

$$E_n = -\frac{R'}{n^2}$$

- We get for n=1 (hydrogen).

$$|\Delta E_{\mathrm{so}}| = \alpha^2 R_H = 13.6\,\mathrm{eV}/137^2 = 0.7\,\mathrm{meV} \equiv 6\,\mathrm{cm}^{-1}$$

- Yet (for a given n) it grows as $Z^2$.

# Speed of light and electric and magnetic fields:

- Imagine two charged wires that are moving at speed v at the same direction:
- There is electrostatic repulsion between them:

$$F_E = k * \frac{2\lambda^2}{d}$$

Coulomb's Constant, equal to $\frac{1}{4\pi\epsilon_0}$

- λ is the charge density

# The magnetic attraction

- If the two wires are moving with speed v:

Electric current, equal to $I$ from above eq.

$$F_B = \frac{(\lambda v)^2 * \mu_0}{2\pi d}$$

- The velocity that these forces are balanced:

$$F_B = \frac{(\lambda v)^2 * \mu_0}{2\pi d} = \frac{2\lambda^2}{4\pi \varepsilon_0 * d} = F_E$$

# cont.

- Is $$\frac{\lambda^2 * v^2 * \mu_0}{2\pi d} = \frac{\lambda^2}{2\pi d * \varepsilon_0}$$

We get $$v^2 * \mu_0 = \frac{1}{\varepsilon_0}$$

and $$v = \sqrt{\frac{1}{\mu_0 * \varepsilon_0}} \qquad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

In the speed of light – a stable wavepocket.

# Spin-orbit coupling beyond the Bohr model

- A nucleus moving around the electron at velocity v results in a magnetic field give by the Biot Savaret eqn.

$$B = \frac{\mu_0}{4\pi} \oint_{\mathrm{loop}} i \, \frac{\mathrm{d}u \times r}{r^3}$$

- du – the orbital path element.
- For simplicity – constant r.

$$\oint i \, \mathrm{d}u = \oint \frac{\mathrm{d}q}{\mathrm{d}t} \, \mathrm{d}u = Ze \frac{\mathrm{d}u}{\mathrm{d}t} = Ze(-v) \, .$$

# Cont.

- We get: $B = -\frac{\mu_0}{4\pi}\frac{Ze}{r^3} v \times r = \frac{\mu_0}{4\pi}\frac{Ze}{r^3} r \times v\,.$
- Coulombs law: The electric field:

$$\mathcal{E} = \frac{Ze}{4\pi\epsilon_0 r^2}\hat{r} = \frac{Ze}{4\pi\epsilon_0 r^3} r$$

- Hat symbol – unit vector.
- We get: $B = \mu_0\epsilon_0\,\mathcal{E} \times v\,.$
- From Maxwells eqn. $\mu_0\epsilon_0 = 1/c^2.$
- We get: $B = \frac{1}{c^2}\mathcal{E} \times v$
- Similar result – non circular orbits and non coulombic interactions in larger atoms.

# The spin orbit interaction:

- The interaction is given by:

$$\Delta E_{\mathrm{so}} = -\mu_{\mathrm{spin}} \cdot B_{\mathrm{orbital}} \, ;$$

- We had:

$$\mu_{\mathrm{spin}} = -g_{\mathrm{s}} \frac{|e|}{2m_{\mathrm{e}}} s = -g_{\mathrm{s}} \frac{\mu_{\mathrm{B}}}{\hbar} s$$

- Substitution: $\Delta E_{\mathrm{so}} = \frac{g_{\mathrm{s}}\mu_{\mathrm{B}}}{\hbar c^2} s \cdot (\mathcal{E} \times v) \, .$
- In a central potential (V function of r only).
- Substituting:

$$\mathcal{E} = \frac{1}{e} \frac{r}{r} \frac{\mathrm{d}V}{\mathrm{d}r}$$

$$v = p/m_{\mathrm{e}}$$

# Cont.

- We get: $$\Delta E_{\mathrm{so}} = \frac{g_{\mathrm{s}}\mu_{\mathrm{B}}}{\hbar c^2 e m_{\mathrm{e}}} \left( \frac{1}{r}\frac{\mathrm{d}V}{\mathrm{d}r} \right) \boldsymbol{s}\cdot(\boldsymbol{r} \times \boldsymbol{p})$$

- Recall: angular momentum – r x p. We get:

$$\Delta E_{\mathrm{so}} = \frac{g_{\mathrm{s}}\mu_{\mathrm{B}}}{\hbar c^2 e m_{\mathrm{e}}} \left( \frac{1}{r}\frac{\mathrm{d}V}{\mathrm{d}r} \right) \boldsymbol{s}\cdot\boldsymbol{l}$$

Electrons that are moving in this speed are relativistic. Including it (thomas precession) gives a factor of 2 reduction. Recall $\mu_{\mathrm{B}} = e\hbar/2m_{\mathrm{e}}$

The final result:

$$\Delta E_{\mathrm{so}} = \frac{g_{\mathrm{s}}}{2} \frac{1}{2c^2 m_{\mathrm{e}}^2} \left( \frac{1}{r}\frac{\mathrm{d}V}{\mathrm{d}r} \right) \boldsymbol{l}\cdot\boldsymbol{s}$$

# For a coulomb field:

$$V = -Ze^2/4\pi\epsilon_0 r$$

and g=2:

$$\Delta E_{\mathrm{so}} = \frac{Ze^2}{8\pi\epsilon_0 c^2 m_{\mathrm{e}}^2}\left(\frac{1}{r^3}\right) l \cdot s$$

Can be used for hydrogenic atoms:

Including e-e repulsion given a more complicated result.

# Evaluation of the spin-orbit energy for hydrogen

- The magnitude of the SO coupling can be written as:

$$\Delta E_{\rm so} = \frac{1}{2c^2 m_{\rm e}^2} \left\langle \frac{1}{r}\frac{{\rm d}V}{{\rm d}r} \right\rangle \langle \boldsymbol{l} \cdot \boldsymbol{s} \rangle$$

- Where g=2 and <> are the expectation value. We calculate:

$$\left\langle \frac{1}{r}\frac{{\rm d}V}{{\rm d}r} \right\rangle = \iiint \psi^*_{nlm} \left( \frac{1}{r}\frac{{\rm d}V}{{\rm d}r} \right) \psi_{nlm} \, r^2 \sin\theta \, {\rm d}r{\rm d}\theta{\rm d}\phi$$

- Which is dependent on r only:

# Cont.

- Only the radial part should be considered.

$$\left\langle \frac{1}{r}\frac{\mathrm{d}V}{\mathrm{d}r} \right\rangle = \int_0^\infty |R_{nl}(r)|^2 \left( \frac{1}{r}\frac{\mathrm{d}V}{\mathrm{d}r} \right) r^2 \mathrm{d}r$$

- In an coulomb field we get:

$$(\mathrm{d}V/\mathrm{d}r)/r \propto 1/r^3$$

- We recall the radial equation:

# Recall

| $n$ | $l$ | $R_{nl}(r)$ |
|---|---|---|
| 1 | 0 | $(Z/a_0)^{\frac{3}{2}} 2 \exp(-Zr/a_0)$ |
| 2 | 0 | $(Z/2a_0)^{\frac{3}{2}} 2 \left(1 - \frac{Zr}{2a_0}\right) \exp(-Zr/2a_0)$ |
| 2 | 1 | $(Z/2a_0)^{\frac{3}{2}} \frac{2}{\sqrt{3}} \left(\frac{Zr}{2a_0}\right) \exp(-Zr/2a_0)$ |
| 3 | 0 | $(Z/3a_0)^{\frac{3}{2}} 2 \left[1 - (2Zr/3a_0) + \frac{2}{3}\left(\frac{Zr}{3a_0}\right)^2\right] \exp(-Zr/3a_0)$ |
| 3 | 1 | $(Z/3a_0)^{\frac{3}{2}} (4\sqrt{2}/3) \left(\frac{Zr}{3a_0}\right) \left(1 - \frac{1}{2}\frac{Zr}{3a_0}\right) \exp(-Zr/3a_0)$ |
| 3 | 2 | $(Z/3a_0)^{\frac{3}{2}} (2\sqrt{2}/3\sqrt{5}) \left(\frac{Zr}{3a_0}\right)^2 \exp(-Zr/3a_0)$ |

$$\int_{r=0}^{\infty} R_{nl}^* R_{nl} r^2 dr = 1.$$

# According to the Bohr model:

We get: $\langle r \rangle = \dfrac{n^2 a_{\rm H}}{Z}$

We make the (crude) conjecture:

$<1/r^3> = Z^3/(n^6\ a_H{}^3)$.

- In fact $\left\langle \dfrac{1}{r^3} \right\rangle = \dfrac{2Z^3}{a^3 n^3 l(l+1)(2l+1)}$

Which is close to the Bohr result when l is close to n

# Recall

- We had:

$$\Delta E_{\mathrm{so}} = \frac{1}{2c^2 m_{\mathrm{e}}^2} \left\langle \frac{1}{r} \frac{\mathrm{d}V}{\mathrm{d}r} \right\rangle \langle \boldsymbol{l} \cdot \boldsymbol{s} \rangle$$

This can be written as:

$$\Delta E_{\mathrm{so}} = C_{nl} \langle \boldsymbol{l} \cdot \boldsymbol{s} \rangle$$

- What is the meaning of the l▪s term?
- We can write:

$$\boldsymbol{j}^2 = (\boldsymbol{l} + \boldsymbol{s})^2 = \boldsymbol{l}^2 + \boldsymbol{s}^2 + 2\boldsymbol{l} \cdot \boldsymbol{s}$$

- Which means:

$$\langle \boldsymbol{l} \cdot \boldsymbol{s} \rangle = \left\langle \frac{1}{2} (\boldsymbol{j}^2 - \boldsymbol{l}^2 - \boldsymbol{s}^2) \right\rangle = \frac{\hbar^2}{2} [j(j+1) - l(l+1) - s(s+1)]$$

# Splitting of the j states:

- For a Coulomb field:

$$\Delta E_{\mathrm{so}} = \frac{Ze^2}{8\pi\epsilon_0 c^2 m_{\mathrm{e}}^2} \left(\frac{1}{r^3}\right) \boldsymbol{l} \cdot \boldsymbol{s}$$

- We replace $1/r^3$ with $\langle 1/r^3 \rangle$

  We have an expression of the quantum numbers:

$$A \frac{j(j+1) - l(l+1) - s(s+1)}{l\left(l + \frac{1}{2}\right)(l+1)}$$

# Cont.

- Recall: $\left\langle \frac{1}{r^3} \right\rangle = \frac{2Z^3}{a^3 n^3 l(l+1)(2l+1)}$
- We get: $A = \frac{Ze^2}{8\pi\varepsilon_0 c^2 m_e^2} \; x \; \frac{Z^3}{a_0^3 n^3} \; x \; \frac{h^2}{4\pi^2}$

$$a_0 = \frac{\hbar}{m_e c}\frac{1}{\alpha} \qquad a_0{}^3 = \frac{h^3}{8\pi^3 m_e{}^3 c^3 \alpha^3}$$

We get: $\frac{Ze^2}{8\pi\varepsilon_0 c^2 m_e^2} \; x \; \frac{Z^3 \, 8\pi^3 m_e{}^3 c^3 \alpha^3}{n^3 h^3} \; x \; \frac{h^2}{4\pi^2}$

# Cont

- A bit algebra:

$$\frac{Z^4 e^2 \pi^2 m_e c \alpha^3}{\varepsilon_0 n^3 h^3} \; x \; \frac{h^2}{4\pi^2} = \frac{\alpha^2 Z^2}{2n^2} \mathrm{X} \frac{2Z^2 e^2 \pi^2 m_e c \alpha}{\varepsilon_0 n h^3} \; x \; \frac{h^2}{4\pi^2}$$

- Taking into account: $\alpha = e^2/2\epsilon_0 hc \approx 1/137$

$$\frac{\alpha^2 Z^2}{2n^2} \mathrm{X} \frac{Z^2 e^4 \pi^2 m_e}{2\varepsilon_0^2 n h^4} \frac{h^2}{4\pi^2} = \frac{\alpha^2 Z^2}{2n^2} \mathrm{X} \frac{Z^2 e^4 m_e}{8\varepsilon_0^2 n h^2}$$

# Recall:

- The gross energy:

$$E_n = -R_{\mathrm{H}} Z^2 / n^2 \quad R_{\mathrm{H}} = 0.9995\, R_\infty hc$$

- and:

$$R_\infty hc = \frac{m_e e^4}{8 \epsilon_0^2 h^2}$$

- We can write: $$E_n \cdot n = \frac{m_e e^4 Z^2}{n 8 \varepsilon_0^2 h^2}$$

Arriving to the final destination:

$$\Delta E_{\mathrm{so}} = -\frac{\alpha^2 Z^2}{2n^2} E_n \frac{n}{l(l+\frac{1}{2})(l+1)} [j(j+1) - l(l+1) - s(s+1)]$$

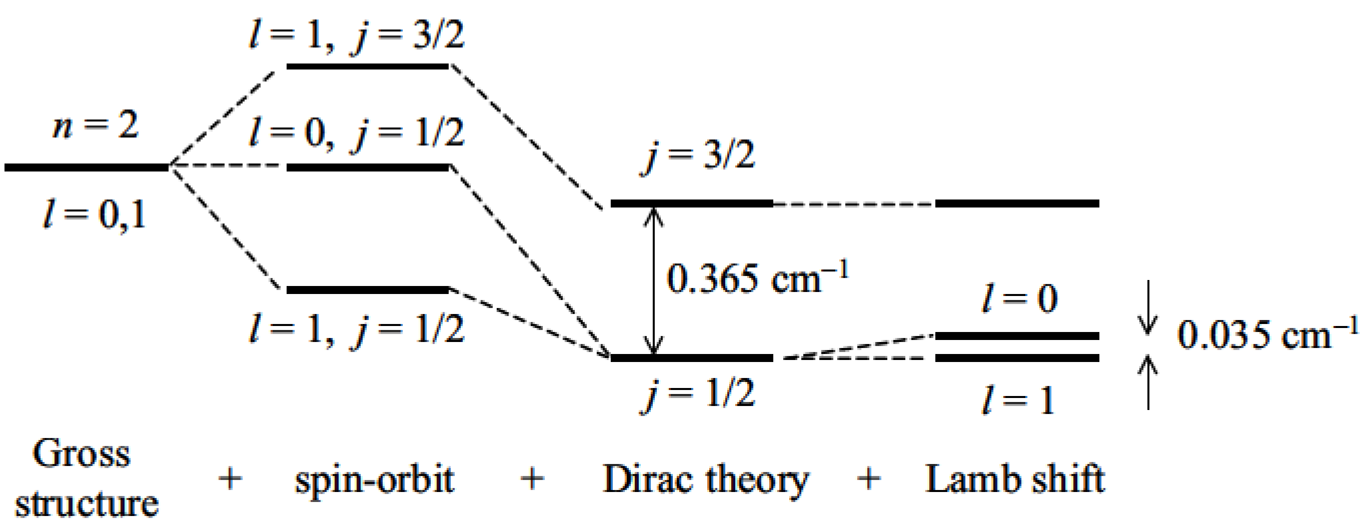
# Recall

- *j takes values of l+1/2 and l-1/2 for l > 1.*

  *SO coupling splits* the two *j states with the same value of l.*

  There are several relativistic corrections.

   - States with the same *n but different l are degenerate. Below - n=2 fine structure of H.*



# Spin-orbit coupling in alkali atoms

- Angular momentum state |*L S  J> of the atom only due to valence electron.*
- *In analogy with hydrogen:*

$$\Delta E_{SO} \propto \langle \mathbf{L} \cdot \mathbf{S} \rangle \propto [J(J+1) - L(L+1) - S(S+1)]$$

- *Valence electron in s state* $\Delta E_{SO}$ *=0.*
- *L=1, S=1/2.* $\mathbf{L} \cdot \mathbf{S} \neq 0$. *J has 2 possible values*

$$J = L \oplus S = L \oplus 1/2 = L \pm 1/2$$

*In a similar way to hydrogen:*

$$\Delta E_{SO} = C\,[J(J+1) - L(L+1) - S(S+1)]$$

# The 2 states.

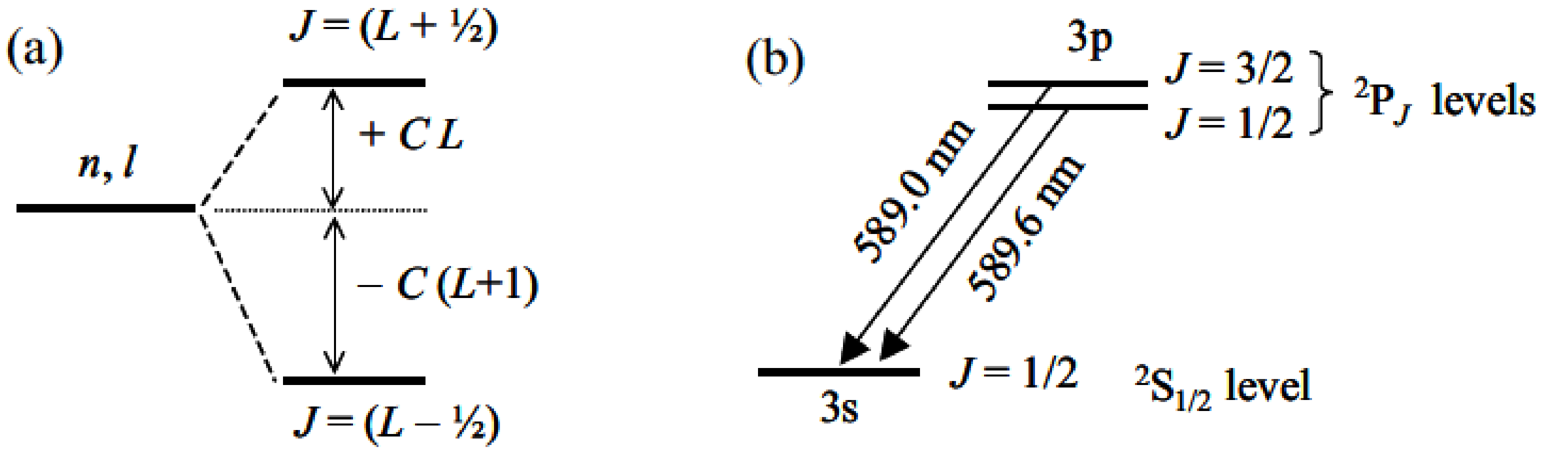
- For the J=L+1/2 state:

$$\Delta E_{\text{so}} = C \left[ (L+\frac{1}{2})(L+\frac{3}{2}) - L(L+1) - \frac{1}{2}\cdot\frac{3}{2} \right] = +CL$$

For the J=L-1/2 state:

$$\Delta E_{\text{so}} = C \left[ (L-\frac{1}{2})(L+\frac{1}{2}) - L(L+1) - \frac{1}{2}\cdot\frac{3}{2} \right] = -C(L+1)$$

The energy diagram of alkali metals and Na.



# The magnitude:

- Compared with gross structure, the fine structure is smaller by $\sim \alpha^2 = 1/137^2$

  Alkali atoms: One electron – with transition: $\mathrm{3p} \rightarrow \mathrm{3s}$

  The ground state is $^2\mathrm{S}_{1/2}$ -no SO coupling.

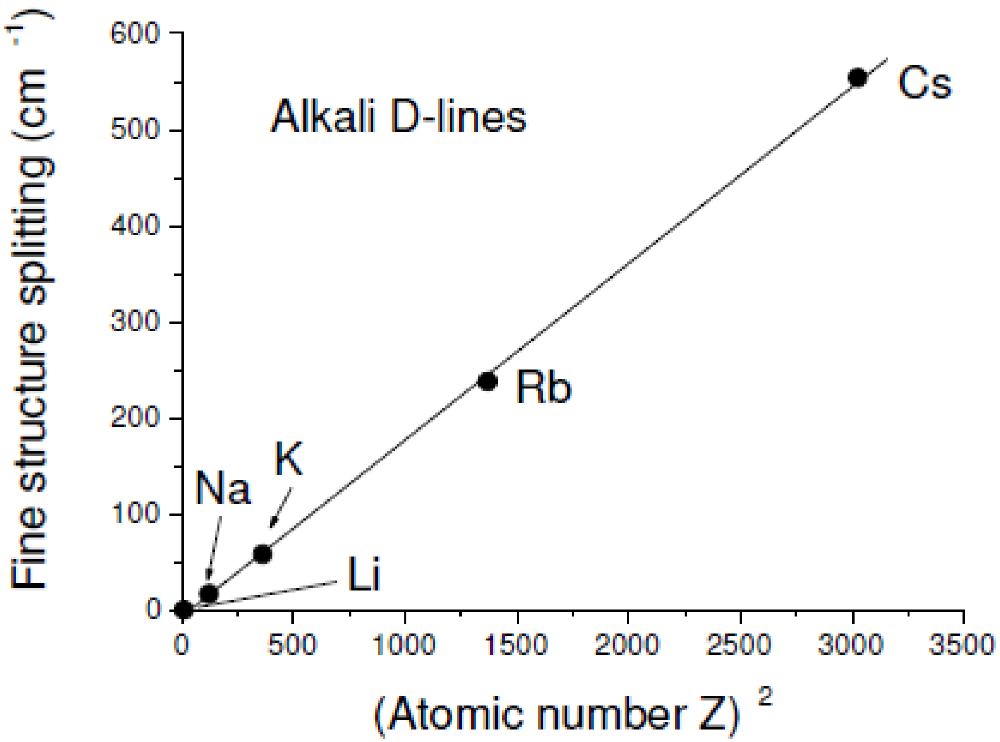
  The two transitions:

  $^2\mathrm{P}_{3/2} \rightarrow {}^2\mathrm{S}_{1/2}$ $\quad$ $^2\mathrm{P}_{1/2} \rightarrow {}^2\mathrm{S}_{1/2}$

  Energy difference: 17cm$^{-1}$

# As was shown already:

- The splitting should be proportional to $Z^2$.



# Spin-orbit coupling in many-electron atoms

- Recall: The residual non spherical electric interactions – couple the electrons to create total S and L angular momenta: These two are coupled to give:

$$J = L + S$$

- J running from L+S to |L-S|.
- the spin-orbit coupling splits the *J states of a* particular *LS-term into fine structure multiplets.*

# As before

- The spin orbit coupling takes the form:

$$\Delta E_{\mathrm{so}} = -\mu_{\mathrm{spin}} \cdot B_{\mathrm{orbital}} \propto \langle \boldsymbol{L} \cdot \boldsymbol{S} \rangle$$

- Which implies:

$$\Delta E_{SO} = C_{LS} \left[ J(J+1) - L(L+1) - S(S+1) \right]$$

- Different J states will have a different energy even with the same L and S

# Nuclear effects in atoms

- The nuclei has small but important affect: the isotope shifts and the hyperfine structure:
- Isotope shifts – mass effects- Recall: in the energy levels of atoms what enters is the reduced mass:

$$\frac{1}{m} = \frac{1}{m_{\mathrm{e}}} + \frac{1}{m_{\mathrm{N}}}$$

- A change in nuclear mass will change the energy.

# Cont.

- Field effects: The electrons have a finite probability to be in the nucleus, and therefore will be sensitive to changes in charge distributions.
- Hyperfine structure: Coupling between the the electron and nucleus angular momenta:

$$\Delta E_{\text{hyperfine}} = -\mu_{\text{nucleus}} \cdot B_{\text{electron}} \propto \langle I \cdot J \rangle$$

# The splitting is small:

- Because the dipole moment of the nucleus is 2000 times smaller than the electron.
- Recall: The gyro magnetic ratio inversely proportional to the mass.
- Total angular momentum:

$$F = I + J$$

- The selection rule:

$$\Delta F = 0, \pm 1 \, ,$$

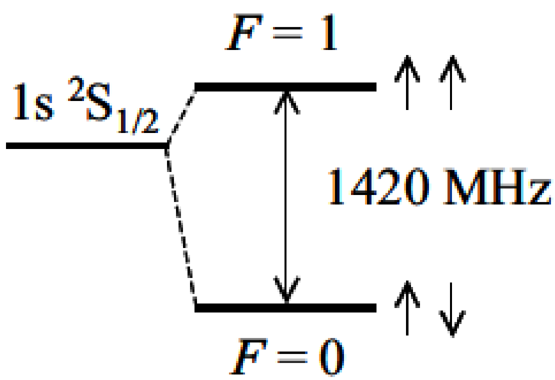
# Examples of hyperfine spectrum:

- The hydrogen ground state is: $1s\ ^2S_{1/2}$ term, (J=1/2).

  Hyperfine quantum number:

  $$F = I \oplus J = 1/2 \oplus 1/2 = 1 \text{ or } 0.$$

  Electron and nuclear spins are parallel (F=1) or antiparallel (F=0).



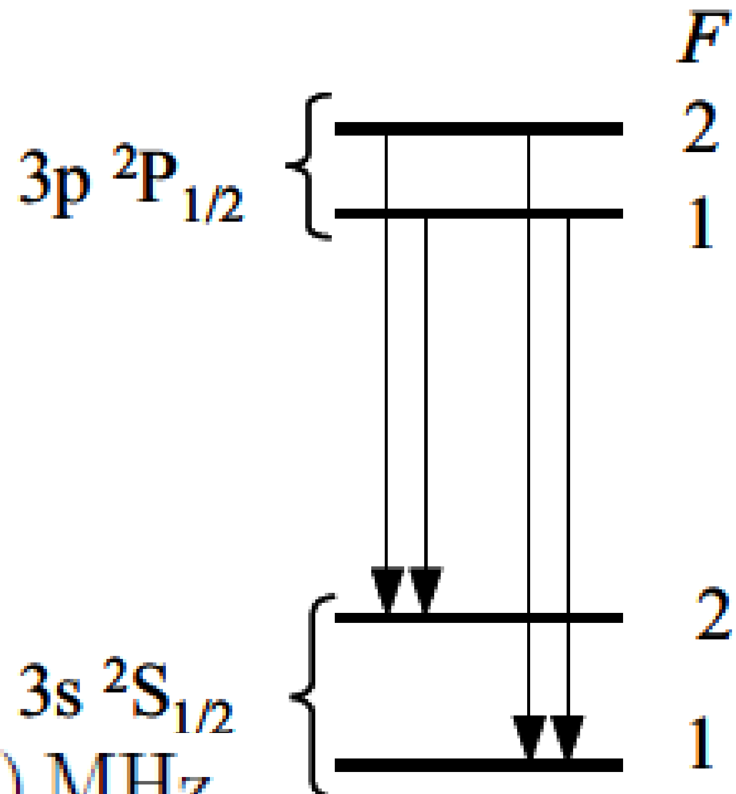

  2 F states split by the hyperfine interaction.

  - The hydrogen song. (NMR)

# Hyperfine structure of sodium D lines.

- D lines originated from 3p→3s transitions. Two transitions split by the SO coupling.
- $D_1$ line: $^2P_{1/2} \rightarrow ^2S_{1/2}$
  Hyperfine spliting of $^2S_{1/2}$ - 1772 MHz. of $^2P_{1/2}$ 190 MHz. Two doublets with relative frequencies:

(0, 190) MHz and (1772, 1962) MHz.



# Cont.

- These splitting – much smaller than the spin orbit splittings of the J states: $5x10^{11}$ Hz.
- The higher energy $D_2$ line:

  We get 6 hyperfine lines

  with relative frequencies:

  0, 34,59 MHz and

  1756, 1772, 1806 MHz.