Prof. E. Palti March 21, 2021

**Particles 1 - Problem Set 1**

(Due: Sunday, April 11, 2021)

# 1 Particle Data Group

Have a look at the Particle Data Group booklet (it is online).

**a)** What are the most likely decay products of $\mu^-$ and $\tau^-$?

**b)** What fractions of top quark decays go to $e\nu_e b$, $\mu\nu_\mu b$ and $\tau\nu_\tau b$.

**c)** What is the mass of the $\eta$-Meson? What is its quark decomposition?

# 2 Position space Feynman diagrams

(This is problem 3.4 in the book of Schwartz)

In section 2.1.6 we studied a scalar field theory with Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{3}\lambda\phi^3 + J\phi \; . \tag{1}$$

**a)** Draw the Feynman diagrams corresponding to the perturbative expansion of the field $\phi(x)$ at order $\lambda^3$ up to $\lambda^5$.

**b)** For each of the Feynman diagrams write down the corresponding term in the expansion of $\phi(x)$.

# 3 Sponaneous Symmetry Breaking

(This is problem 3.5 in the book of Schwartz)

A simple classical example of spontaneous symmetry breaking is described by the Lagrangian for a scalar with a negative mss term

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 \; . \tag{2}$$

**a)** How many constants $c$ can you find for which $\phi(x) = c$ is a solution to the equations of motion? Which solution has the lowest energy (the ground state)?

**b)** The Lagrangian has a symmetry $\phi \to -\phi$. Show that this symmetry is not respected by the ground state. We say that the vacuum expectation value of $\phi$ is $c$ and write $<\phi>= c$. In this vacuum, the $\mathbb{Z}_2$ symmetry $\phi \to -\phi$ is spontaneously broken.

**c)** Write $\phi(x) = c + \pi(x)$ and substitute back into the Lagrangian. Show that now $\pi = 0$ is a solution to the equations of motion. How does $\pi$ transform under the $\mathbb{Z}_2$ symmetry $\phi \to -\phi$? Show that this is a symmetry of the Lagrangian of $\pi$.

# 4 Field operators and commutation relations

Consider the expansions for the field $\phi(\mathbf{x})$ and its conjugate $\Pi(\mathbf{x})$,

$$\phi(\mathbf{x}) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{\sqrt{2\omega_{\mathbf{p}}}} \left( a_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{x}} + a^\dagger_{\mathbf{p}} e^{-i\mathbf{p}\cdot\mathbf{x}} \right) , \tag{3}$$

$$\Pi(\mathbf{x}) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} (-i) \sqrt{\frac{\omega_{\mathbf{p}}}{2}} \left( a_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{x}} - a^\dagger_{\mathbf{p}} e^{-i\mathbf{p}\cdot\mathbf{x}} \right) , \tag{4}$$

where $\omega_{\mathbf{p}}^2 = |\mathbf{p}|^2 + m^2$.

**a)** In quantum field theory, the field is promoted to an operator. Calculate the momentum projection of the action on $\phi(\mathbf{x})$ on the vacuum $<\mathbf{p}|\phi(\mathbf{x})|0>$. Compare this to the projection of a position state onto a momentum state in 1-particle quantum mechanics, and use this to answer: what is the physical meaning of the action of $\phi(\mathbf{x})$ on the vacuum?

**b)** Show that the momentum space commutation relations

$$\left[a_{\mathbf{k}}, a^\dagger_{\mathbf{p}}\right] = (2\pi)^3 \delta^{(3)}(\mathbf{k} - \mathbf{p}) \ , \ \ [a_{\mathbf{k}}, a_{\mathbf{p}}] = 0 \ , \ \ \left[a^\dagger_{\mathbf{k}}, a^\dagger_{\mathbf{p}}\right] = 0 \ , \tag{5}$$

imply the spacetime commutation relations

$$[\phi(\mathbf{x}), \Pi(\mathbf{y})] = i\delta^{(3)}(\mathbf{x} - \mathbf{y}) \ , \ \ [\phi(\mathbf{x}), \phi(\mathbf{y})] = 0 \ , \ \ [\Pi(\mathbf{x}), \Pi(\mathbf{y})] = 0 \ . \tag{6}$$

# 5 Spinor sum identities

(This is problem 11.2 in the book of Schwartz)

Consider the solutions to the Dirac equation from section 2.2.3.

**a)** Show the spin-sum identities

$$\sum_s u_s(p)\bar{u}_s(p) = \gamma \cdot p + m\,\mathbb{1}\,, \qquad (7)$$

$$\sum_s v_s(p)\bar{v}_s(p) = \gamma \cdot p - m\,\mathbb{1}\,. \qquad (8)$$

**b)** Show the identity

$$\bar{u}_s(p)\gamma^\mu u_{s'}(p) = 2\delta_{ss'}p^\mu\,. \qquad (9)$$