Prof. E. Palti April 21, 2021

Particles 1 - Problem Set 2

(Due: Thursday, May 6th, 2021)

1 Rutherford Scattering

(This is working through section 13.4 in the book of Schwartz)

We would like to consider Rutherford scattering: an electron scattering off a proton $e^-p^+ \to e^-p^+$. We will treat the proton as point-like.

a)* **(Optional)** Recall from non-relativistic quantum mechanics that in the Born approximation the cross-section for this process is related to the Fourier transform of the Coulomb potential

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Born}} = \frac{m_e^2}{4\pi^2}\tilde{V}\left(\mathbf{k}\right)^2 \ . \tag{1}$$

Here $\mathbf{k}$ is the momentum transfer during the scattering $\mathbf{k} = \mathbf{p}_i - \mathbf{p}_f$, which satisfies $|\mathbf{k}| = 2\,|\mathbf{p}_i|\sin\left(\frac{\theta}{2}\right)$. Here $\mathbf{p}_i$ is the initial spatial momentum, $\mathbf{p}_i$ the final one, and they satisfy $|\mathbf{p}_i| = |\mathbf{p}_f|$.

Show the this gives the non-relativistic expression

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Born}} = \frac{m_e^2 e^4}{64\pi^4\,|\mathbf{p}_i|^4\sin^4\left(\frac{\theta}{2}\right)} \ . \tag{2}$$

b) Treat the proton as a fundamental point-like particle (with convention that p^+ is the particle and p^- the anti-particle.) Draw the Feynman diagram for this process. Using the QED Feynman rules, write the associated Matrix element.

c) Show that the unpolarized scattering, so averaging over initial spins, gives

$$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2 = \frac{2e^4}{t^2}\left[u^2 + s^2 + 4t\left(m_e^2 + m_p^2\right) - 2\left(m_e^2 + m_p^2\right)^2\right] \ . \tag{3}$$

d) Calculate the differential cross-section in the centre–of-mass frame in the limit $m_p \gg m_e$. Show that this reduces to Rutherford's formula (2) in the non-relativistic limit where the electron's velocity is small $v \ll 1$.

2 Gauge invariance of Non-Abelian kinetic terms

The kinetic terms for an $SU(2)$ gauge field are

$$\mathcal{L}_{\text{kin}} = -\frac{1}{4}\sum_{a=1}^{3} F^a_{\mu\nu}F^{a,\mu\nu} \ , \tag{4}$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc}A^b_\mu A^c_\nu \ . \tag{5}$$

a) Show that under an infinitesimal gauge transformation

$$A^a_\mu \to A^a_\mu + \frac{1}{g}\partial_\mu \alpha^a - \epsilon^{abc}\alpha^b A^c_\mu \ , \tag{6}$$

the field strength transforms as

$$\delta F^a_{\mu\nu} = -g\epsilon^{abc}\alpha^b F^c_{\mu\nu} \ . \tag{7}$$

b) Show that while the field strength is not gauge invariant, the kinetic term in the Lagrangian is gauge invariant.