Homework 1

Question 1

Given $\vec{G}\left(\vec{x}\right) = \vec{x}e^{-\alpha r}$,

1. Find $\vec{\nabla} \cdot \vec{G}$.
2. Sketch $\vec{\nabla} \cdot \vec{G}$ as a function of r.
3. Find $\int_V \vec{\nabla} \cdot \vec{G} \mathrm{d}V$, where V is the volume of a sphere with radius a centered at the origin.
4. Find the answer to (3) in the limit $a \to \infty$.

Question 2

In Cartesian, spherical and cylindrical coordinates, find

$$\vec{\nabla} \cdot \vec{r}, \vec{\nabla}\left(\vec{C} \cdot \vec{r}\right), \vec{\nabla} \times \vec{r},$$

where $\vec{C}$ is some constant vector and $\vec{r}$ is the position vector.

Question 3

Solve Griffith's 1.53-1.56:

Problem 1.53 Check the divergence theorem for the function

$$\mathbf{v} = r^2 \cos\theta\, \hat{\mathbf{r}} + r^2 \cos\phi\, \hat{\boldsymbol{\theta}} - r^2 \cos\theta \sin\phi\, \hat{\boldsymbol{\phi}},$$

using as your volume one octant of the sphere of radius R (Fig. 1.48). Make sure you include the *entire* surface. [*Answer:* $\pi R^4/4$]

Problem 1.54 Check Stokes' theorem using the function $\mathbf{v} = ay\,\hat{\mathbf{x}} + bx\,\hat{\mathbf{y}}$ (a and b are constants) and the circular path of radius R, centered at the origin in the xy plane. [*Answer:* $\pi R^2(b-a)$]

Problem 1.55 Compute the line integral of

$$\mathbf{v} = 6\,\hat{\mathbf{x}} + yz^2\,\hat{\mathbf{y}} + (3y+z)\,\hat{\mathbf{z}}$$

along the triangular path shown in Fig. 1.49. Check your answer using Stokes' theorem. [*Answer:* 8/3]

Problem 1.56 Compute the line integral of

$$\mathbf{v} = (r\cos^2\theta)\,\hat{\mathbf{r}} - (r\cos\theta\sin\theta)\,\hat{\boldsymbol{\theta}} + 3r\,\hat{\boldsymbol{\phi}}$$

around the path shown in Fig. 1.50 (the points are labeled by their Cartesian coordinates). Do it either in cylindrical or in spherical coordinates. Check your answer, using Stokes' theorem. [*Answer:* $3\pi/2$]

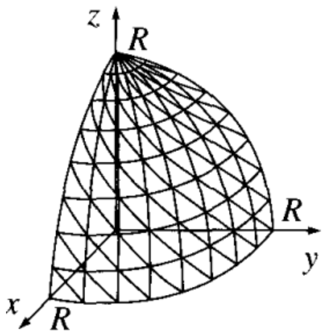

Figure 1.48

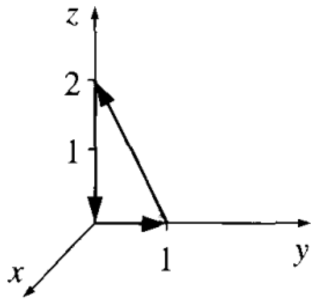

Figure 1.49

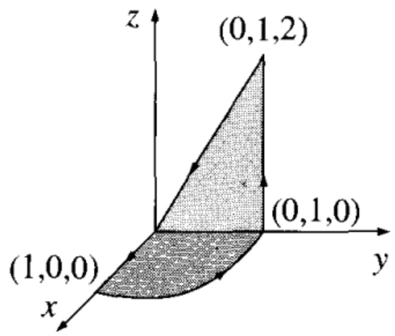

Figure 1.50

Question 4

Compute the Gradient and the Divergence in spherical coordinates. **Hint:** Use the differential form of the gradient of a scalar function, $\mathrm{d}f = \vec{\nabla} f \cdot \mathrm{d}\vec{r}$ and recall that $\mathrm{d}\vec{r}$ is **not** $(\mathrm{d}r, \mathrm{d}\theta, \mathrm{d}\varphi)$.

Question 5

The time-averaged potential of a neutral hydrogen atom is given by

$$\Phi = \frac{q}{4\pi} \frac{e^{-\alpha r}}{r} \left(1 + \frac{\alpha r}{2}\right),$$

where in our units $\alpha = 2/a_0$, and a_0 is the Bohr radius. Find the distribution of charge that gives this potential. Interpret your result physically.