Homework 8

Question 1

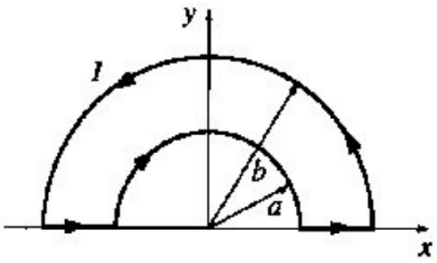
Suppose you take a plastic ring of radius a and glue charge on it, so that the line charge density is $\lambda = \lambda_0 |\sin(\theta/2)|$. Then you spin the loop about its axis at an angular velocity ω. Find the (exact) scalar and vector potentials at the center of the ring.

Question 2

A piece of wire bent into a loop, as shown in the figure, carries a current that increases linearly with time:

$$I(t) = kt \tag{1}$$

Calculate the retarded vector potential A at the center. Find the electric field at the center. Why does this (neutral) wire produce an electric field? (Why can't you determine the magnetic field from this expression for A?)



Question 3

A point particle with charge q_1 is moving along the x axis towards the origin in a trajectory

$$x\left(t\right)=\sqrt{b^2+c^2t^2}.$$

At time $t=0$ the particle gets as close as he can get to the origin, $x=b$, and then begins to move away. There is an additional particle with charge q_2 that is held fixed at the origin ($x=0$) at all times.

1. Find the retarded time for the charge q_1 for an observer at the origin. **Hint**: You need to solve a non-linear equation for the retarded time.

2. Find the electric and magnetic fields at the origin from the charge q_1. Use the field related to the Lienard-Weichert potentials.

3. Show that the force acting on the charge q_2 from q_1 is equal to

$$\vec{F}_{1\to2}=-\frac{4q_1q_2b^2}{\left(b^2+c^2t^2\right)^2}\hat{\mathbf{x}},$$

   for $t>0$. Assume that $\vec{F}_{1\to2}\left(t<0\right)=0$.

4. Find the electric and magnetic fields at the position of charge q_1 from the charge q_2, as a function of time, in the frame of reference where q_2 is static.

5. What is the force $\vec{F}_{2\to1}$ from the charge q_2 on q_1?

6. Is there a frame of reference where $\vec{F}_{2\to1}=\vec{F}_{1\to2}$? If so, is it inertial?