# Homework 9

## Question 1

**Radiation from Rotating Ring**
An insulating circular ring (radius $b$) lies in the xy plane, centered at the origin. It carries a linear charge density $\lambda = \lambda_0 \sin\theta$, where $\lambda_0$ is constant and $\theta$ is the usual azimuthal angle. The ring is now set spinning at a constant angular velocity $\omega$ about the $z$ axis. Calculate the power radiated.

**Hint**: The Poynting vector of a dipole is given by

$$\vec{S} = \frac{1}{4\pi c^3 r^2} \left| \hat{\mathbf{n}} \times \ddot{\vec{p}}\,(t_{\rm ret}) \right|^2 \hat{\mathbf{n}}.$$

Note that you are not asked to take a time average of the radiation in this case.

## Question 2

In this question you will find the radiation of an oscillating electric dipole.

Imagine two tiny metal spheres separated by a distance $d$ and connected by a fine wire (see figure). At time $t$ the charge on the upper sphere is $q(t)$ and on the lower sphere it is $-q(t)$. Suppose we drive the charge back and forth through the wire with angular frequency $\omega$, such that

$$q(t) = q_0 \cos(\omega t),$$

resulting in an oscillating electric dipole

$$\vec{p}\,(t) = p_0 \cos(\omega t)\,\hat{\mathbf{z}} = q_0 d \cos(\omega t)\,\hat{\mathbf{z}}.$$

1. The retarded potential is

$$\Phi(t, \vec{r}) = \frac{1}{4\pi} \left[ \frac{q_0 \cos(\omega(t - r_+/c))}{r_+} - \frac{q_0 \cos(\omega(t - r_-/c))}{r_-} \right].$$

   Write $r_\pm$ explicitly in terms of $r, d$ and $\theta$.

2. This *physical* dipole becomes a *perfect* dipole when the separation distance is very small.

Approximate $d \ll r$ (but $d > 0!$) and show that to first order in $d$,

$$\cos\left(\omega\left(t - r_{\pm}/c\right)\right) \cong \cos\left(\omega\left(t - r/c\right)\right)\cos\left(\frac{\omega d}{2c}\cos\theta\right) \mp \sin\left(\omega\left(t - r/c\right)\right)\sin\left(\frac{\omega d}{2c}\cos\theta\right).$$

3. In the perfect dipole limit, we further have

$$d \ll \frac{2\pi c}{\omega} = \lambda.$$

Under this assumption, show that the potential of an oscillating perfect dipole is

$$\Phi\left(t, r, \theta\right) \cong \frac{p_0 \cos\theta}{4\pi r}\left[-\frac{\omega}{c}\sin\left(\omega\left(t - r/c\right)\right) + \frac{1}{r}\cos\left(\omega\left(t - r/c\right)\right)\right]. \tag{1}$$

4. To find the radiation of the dipole, not every piece of the potential is useful; Only the fields which survive large at distances for the source, in the *radiation zone*

$$r \gg \frac{c}{\omega} = \frac{\lambda}{2\pi},$$

contribute to the radiation. Write the scalar potential in this case.

5. The vector potential is determined by the current flowing in the wire,

$$I\left(t\right) = \frac{\mathrm{d}q}{\mathrm{d}t}\hat{\mathbf{z}} = -q_0\omega\sin\left(\omega t\right)\hat{\mathbf{z}}.$$

Show that the vector potential can be similarly written in the form

$$\vec{A}\left(t, \vec{r}\right) = -\frac{p_0\omega}{cr}\sin\left[\omega\left(t - r/c\right)\right]\mathrm{d}z\hat{\mathbf{z}}.$$

**Hint**: Approximate the integral on the vector potential to first order in $d$, such that the integration is replaced by a factor of $d$ and the integrand is evaluated at the center.

6. Find the electric and magnetic fields under the approximation $r \gg \lambda$.

7. Find the time average Poynting vector, where the average is taken over a complete cycle.

8. Using the relation between intensity and power,

$$\langle P\rangle_T = \int\left\langle\vec{S}\right\rangle_T \cdot \mathrm{d}\vec{A},$$

where $A$ is the surface over a sphere, find the radiation power.

# Question 3

**Radiation for Bohr's Atom**

In Bohr's theory of hydrogen, the electron in its ground state was supposed to travel in a circle of radius $5 \times 10^{-11}$, held in orbit by the Coulomb attraction of the proton.

According to classical electrodynamics, this electron should radiate, and hence spiral in to the nucleus. Show that $v \ll c$ for most of the trip (so you can use the Larmor formula), and calculate the lifespan of Bohr's atom. (Assume each revolution is essentially circular.)

# Question 4

1. Suppose an electron decelerates at a constant rate from some initial velocity $v_0 \ll c$ down to zero. Calculate what fraction of the initial kinetic energy is lost to radiation (the rest is absorbed by whatever mechanism keeps the acceleration constant).

2. To get a sense of the numbers involved, suppose that the initial velocity is thermal ($\sim 10^5\,[m/s]$) and that the distance the electron travels until it stops is 30 ångström ($10^{-10}[m]$). What can you conclude about radiation losses for the electrons in an ordinary conductor?