

Particles 1 - Problem Set 1

(Due: November 29, 2022)

1 Particle Data Group

Have a look at the Particle Data Group booklet (it is online).

- a) What are the most likely decay products of μ^- and τ^- ?
- b) What fractions of top quark decays go to $e\nu_e b$, $\mu\nu_\mu b$ and $\tau\nu_\tau b$.
- c) What is the mass of the η -Meson? What is its quark decomposition?

2 Position space Feynman diagrams

(This is problem 3.4 in the book of Schwartz)

In section 2.1.6 we studied a scalar field theory with Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{3}\lambda\phi^3 + J\phi . \quad (1)$$

- a) Draw the Feynman diagrams corresponding to the perturbative expansion of the field $\phi(x)$ at order λ^3 up to λ^5 .
- b) For each of the Feynman diagrams write down the corresponding term in the expansion of $\phi(x)$.

3 Spontaneous Symmetry Breaking

(This is problem 3.5 in the book of Schwartz)

A simple classical example of spontaneous symmetry breaking is described by the Lagrangian for a scalar with a negative mass term

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 . \quad (2)$$

a) How many constants c can you find for which $\phi(x) = c$ is a solution to the equations of motion? Which solution has the lowest energy (the ground state)?

b) The Lagrangian has a symmetry $\phi \rightarrow -\phi$. Show that this symmetry is not respected by the ground state. We say that the vacuum expectation value of ϕ is c and write $\langle \phi \rangle = c$. In this vacuum, the \mathbb{Z}_2 symmetry $\phi \rightarrow -\phi$ is spontaneously broken.

c) Write $\phi(x) = c + \pi(x)$ and substitute back into the Lagrangian. Show that now $\pi = 0$ is a solution to the equations of motion. How does π transform under the \mathbb{Z}_2 symmetry $\phi \rightarrow -\phi$? Show that this is a symmetry of the Lagrangian of π .

4 Field operators and commutation relations

Consider the expansions for the field $\phi(\mathbf{x})$ and its conjugate $\Pi(\mathbf{x})$,

$$\phi(\mathbf{x}) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{\sqrt{2\omega_{\mathbf{p}}}} (a_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{x}} + a_{\mathbf{p}}^\dagger e^{-i\mathbf{p}\cdot\mathbf{x}}) , \quad (3)$$

$$\Pi(\mathbf{x}) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} (-i) \sqrt{\frac{\omega_{\mathbf{p}}}{2}} (a_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{x}} - a_{\mathbf{p}}^\dagger e^{-i\mathbf{p}\cdot\mathbf{x}}) , \quad (4)$$

where $\omega_{\mathbf{p}}^2 = |\mathbf{p}|^2 + m^2$.

a) In quantum field theory, the field is promoted to an operator. Calculate the momentum projection of the action on $\phi(\mathbf{x})$ on the vacuum $\langle \mathbf{p} | \phi(\mathbf{x}) | 0 \rangle$. Compare this to the projection of a position state onto a momentum state in 1-particle quantum mechanics, and use this to answer: what is the physical meaning of the action of $\phi(\mathbf{x})$ on the vacuum?

b) Show that the momentum space commutation relations

$$[a_{\mathbf{k}}, a_{\mathbf{p}}^\dagger] = (2\pi)^3 \delta^{(3)}(\mathbf{k} - \mathbf{p}) , \quad [a_{\mathbf{k}}, a_{\mathbf{p}}] = 0 , \quad [a_{\mathbf{k}}^\dagger, a_{\mathbf{p}}^\dagger] = 0 , \quad (5)$$

imply the spacetime commutation relations

$$[\phi(\mathbf{x}), \Pi(\mathbf{y})] = i\delta^{(3)}(\mathbf{x} - \mathbf{y}) , \quad [\phi(\mathbf{x}), \phi(\mathbf{y})] = 0 , \quad [\Pi(\mathbf{x}), \Pi(\mathbf{y})] = 0 . \quad (6)$$

5 Spinor sum identities

(This is problem 11.2 in the book of Schwartz)

Consider the solutions to the Dirac equation from section 2.2.3.

a) Show the spin-sum identities

$$\sum_s u_s(p) \bar{u}_s(p) = \gamma \cdot p + m \mathbb{1} , \quad (7)$$

$$\sum_s v_s(p) \bar{v}_s(p) = \gamma \cdot p - m \mathbb{1} . \quad (8)$$

b) Show the identity

$$\bar{u}_s(p) \gamma^\mu u_{s'}(p) = 2\delta_{ss'} p^\mu . \quad (9)$$