The physics of the atmosphere & ocean

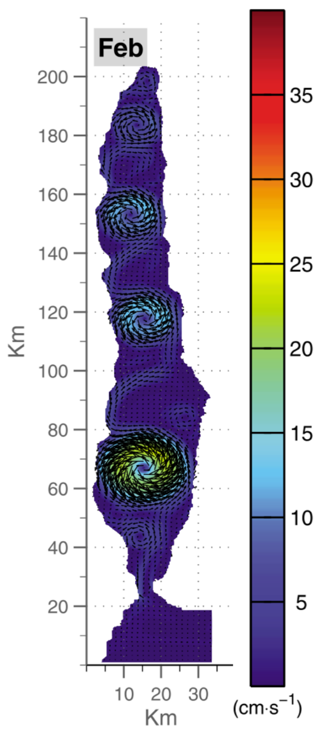
Exercise 2

Coriolis force and its significance:

1) Can the Coriolis force affect the outcome of a baseball tournament? In a tournament, a ball is thrown horizontally 25 m/s. By how much will it be deflected by the Coriolis force? You can assume the latitude is $30°N$.

2) Imagine that Concorde is (was) flying at speed u from New York to London along a latitude circle. The deflecting force due to the Coriolis effect is toward the south. By lowering the left wing ever so slightly, the pilot (or perhaps more conveniently the computer on board) can balance this deflection. Draw a diagram of the forces—gravity, uplift normal to the wings, and Coriolis—and use it to deduce that the angle of tilt, γ, of the aircraft from the horizontal required to balance the Coriolis force is $\tan\gamma = \frac{2\Omega \sin\phi u}{g}$ where Ω is the Earth's rotation, the latitude is ϕ and gravity is g. You can assume that the Coriolis force is negligible compared to Earth's gravity. If $u = 600 \text{ m s}^{-1}$, insert typical numbers to compute the angle. What analogies can you draw with atmospheric circulation?

3) Geostrophic balance:
Use the attached velocity map of the Gulf of Eilat (taken from numerical modeling) to give an estimate of the surface height difference between the middle and the outer edge of the southern gyre. Assume geostrophic balance and that the water column is fully mixed. The latitude in the upper northern Gulf (Eilat) is 29.5°N and the mean velocity of the gyre is 25 cm/s.



4) Potential Vorticity:
Use the following frictionless and barotropic equations

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - fv = -g\frac{\partial h}{\partial x}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + fu = -g\frac{\partial h}{\partial y}$$

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv) = 0$$

to derive the vorticity equation:

$$\frac{D}{Dt}\left(\frac{\xi + f}{h}\right) = 0$$

Hint: Find the z component of the curl of the 2 horizontal velocity equations and identify the vorticity terms and the full derivatives.

5) Energy conservation:
Using the shallow water equations with flat bottom, prove that the total energy [defined as $E = \frac{1}{2}\iint (h(u^2 + v^2) + gh^2)dxdy$] is preserved. You may assume periodic boundary conditions in the x and y directions.