The physics of the atmosphere & ocean

Exercise 3

Ekman layer:

1) It is observed that fragments of tea leaves at the bottom of a stirred tea cup conglomerate toward the center. Explain this phenomenon with Ekman-layer dynamics. Also explain why the tea leaves go to the center irrespectively of the direction of stirring (clockwise or counterclockwise).

Surface Ekman layer:

2) (a) Assume geostrophic flow below a surface Ekman layer. Wind blowing over the surface exerts stress at the ocean surface, forcing surface currents that decay to the geostrophic flow at the interior (i.e., for $z \to -\infty, u = \bar{u}$ & $v = \bar{v}$). The boundary conditions at the surface (z=0) are: $\rho_0 \nu \frac{\partial u}{\partial z} = \tau^x, \rho_0 \nu \frac{\partial v}{\partial z} = \tau^y$, where $\vec{\tau} = (\tau^x, \tau^y)$ is the wind stress. Show that velocity in the surface layer

$$u = \bar{u} + \frac{\sqrt{2}}{\rho_0 f d} e^{z/d} \left[\tau^x \cos\left(\frac{z}{d} - \frac{\pi}{4}\right) - \tau^y \sin\left(\frac{z}{d} - \frac{\pi}{4}\right)\right]$$

$$v = \bar{v} + \frac{\sqrt{2}}{\rho_0 f d} e^{z/d} \left[\tau^x \sin\left(\frac{z}{d} - \frac{\pi}{4}\right) + \tau^y \cos\left(\frac{z}{d} - \frac{\pi}{4}\right)\right]$$

where $d = \sqrt{\frac{2\nu}{f}}$.

(b) What will be the vertical velocity at the lower boundary of the layer, resulted from the Ekman transport in the layer? (in terms of the geostrophic velocities at the layer below the Ekman layer.)

(c) What is the angle between the wind direction and the surface current vector and what is the angle between the wind direction and the transport vector?

3) Assume that the atmospheric Ekman layer over the earth's surface at latitude 45°N can be modeled with an eddy viscosity $\nu_E = 10\ m^2/s$. If the geostrophic velocity above the layer is 10 m/s and uniform, what is the vertically integrated flow across the isobars (pressure contours)? Is there any vertical velocity?

4) The variation of the Coriolis parameter with latitude can be approximated as $f = f_0 + \beta_0 y$, where y is the northward coordinate. Using this, show that the vertical velocity below the surface Ekman layer of the ocean is given by

$$\bar{w}(z) = \frac{1}{\rho_0}\left[\frac{\partial}{\partial x}\left(\frac{\tau^y}{f}\right) - \frac{\partial}{\partial y}\left(\frac{\tau^x}{f}\right)\right] - \frac{\beta_0}{f}\int_z^0 \bar{v} dz$$

Where τ^x and τ^y are the zonal and meridional wind-stress components, respectively, and $\bar{v}$ is the meridional velocity in the geostrophic interior below the Ekman layer.