Exercise 4

Barotropic Waves

1. Calculate the Rossby radius of deformation for (i) latitude 35°N and depths of 100, 500, 1000 and 4000m; (ii) depth of 2000m and latitudes 5°,15°,30°,40°,50°,65°N. Plot $R(H)$, R(latitude).

2. The Yellow Sea between China and Korea (mean latitude: 37°N) has an average depth of 50 m and a coastal perimeter of 2600 km. How long does it take for a Kelvin wave to go around the shores of the Yellow Sea?

3. Because the Coriolis parameter vanishes along the Equator, it is usual in the study of tropical processes to write $f = f_0 + \beta y$, where y is the distance measured from the Equator (positive northward). The linear wave equations then take the form:

$$\frac{\partial u}{\partial t} - \beta_0 y v = -g\frac{\partial \eta}{\partial x}$$
$$\frac{\partial v}{\partial t} + \beta_0 y u = -g\frac{\partial \eta}{\partial y}$$
$$\frac{\partial \eta}{\partial t} + H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$$

where u and v are the zonal and meridional velocity components, η is the surface displacement, g is gravity constant, and H is the ocean depth at rest. Solve the above equations. Explore the possibility of a wave traveling zonally with no meridional velocity.
- At which speed does this wave travel and in which direction?
- Is it trapped along the Equator?
- If so, what is the trapping distance?
- Does this wave bear any resemblance to a mid-latitude wave (f_0 not zero)?

4. Topographic waves:
   a) Just as small variations in the Coriolis parameter can support Rossby (planetary) waves, so can small variations in depth. Assume f-plane with $h(y)=H_0+\alpha y$ where H_0 is a mean reference depth and α is the bottom slope. $\alpha L/ H_0 << 1$, where L is the horizontal length scale of the motion. Follow a similar procedure to that used in class to derive Rossby waves, and find a dispersion relation for topographic waves.
   b) How topographic waves propagate in the Northern hemisphere regarding the shallow water side (on their right or left?)
   c) Try to explain the analogy between Rossby (planetary) and topographic waves, based on the potential vorticity?

   - The Bottom slope gives rise to a new term in the continuity equation
   - $\frac{1}{1+x} \approx 1 - x$ for x<<1

- Assume that the small parameter is $\varepsilon = \frac{\alpha R}{H_0}$, where $R = \frac{\sqrt{gH_0}}{f_0}$.
- Use the following scaling:

$$t = \frac{\tilde{t}\sqrt{H_0}}{\alpha\sqrt{g}} \quad (u,v) = \sqrt{gH_0}(\tilde{u},\tilde{v}) \quad (x,y) = R(\tilde{x},\tilde{y}) \quad \eta = H_0\tilde{\eta}$$

where the variables with the tilde (~) are non-dimensional.

5. Poincare waves on a constant current:
Assume that a wave is "riding" on a constant zonal current *U* such that the zonal velocity becomes $u(x,y,t)=U+u'(x,y,t)$ and that u', v are sufficiently small such that the nonlinear terms can be ignored.
a) Start from the nonlinear shallow water equations

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - fv = -g'\frac{\partial h}{\partial x},$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + fu = -g'\frac{\partial h}{\partial y},$$

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv) = 0,$$

Write down the linearized equations under the above assumptions.
b) Now find a general wave solution for the homogeneous equations and a particular solution for the full system. What are the dispersion relation and the zonal phase speed? Explain the zonal phase speed physically.
c) Find u, v and η. What is the relation between them?