Exercise 6

Stratification and internal Waves

1) a) A scuba diver dives with a (flexible) balloon filled with air from the surface to a depth H. What will be the volume of the balloon at this depth relative to its original volume? You may assume that the water density is constant and that the temperature within the balloon remain unchanged.
   b) Repeat (a) when assuming that the ocean is linearly stratified (by salt) with buoyancy frequency N. You can still assume that the air temperature inside the balloon remains constant.
   c) What will be the density of the balloon at a depth of 10km? Describe the vertical motion of the balloon at this depth.

2) Internal waves are generated along the coast of Norway by the M_2 surface tide (period of 12.42 h). If the buoyancy frequency N is $2\text{x}10^{-3}$ s^{-1}, at which possible angles can the energy propagate with respect to the horizontal?

3) Internal waves including rotation:
   a) Derive the dispersion relation (in terms of wave numbers and N) of internal gravity waves in the presence of rotation, assuming $f < N$.
   b) Show that the frequency of these waves must always be higher than f but lower than N.
   c) Find the velocities and the density.
   d) Compare vertical phase speed to vertical group velocity.

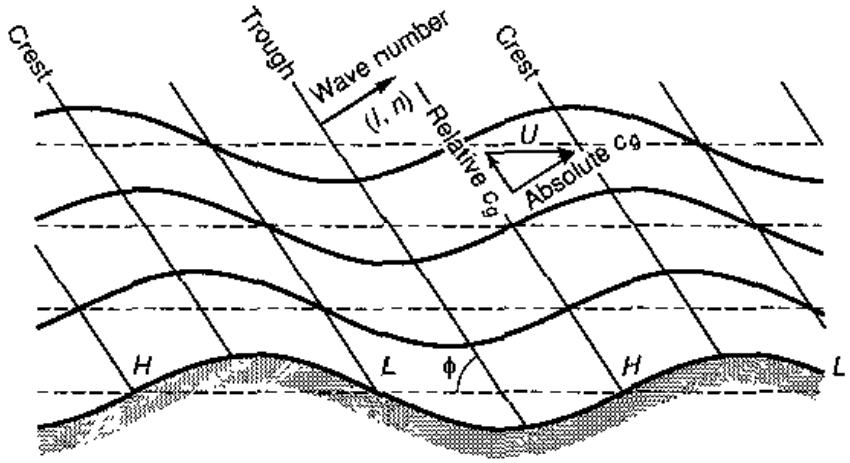
4) Lee waves:
   a) Show that the title angel between the waves fronts (lines joining crests, see Figure) and the horizontal, ϕ, is given by:

$$\sin\phi = \frac{k_x U}{N},$$

   where k_x is the wave number of the periodic terrain, U is the wind velocity, and N is the stratification frequency.
   b) Calculate the group velocity in the x, z direction is:

$$c_{gx} = U\sin^2\phi\,;\; c_{gz} = U\sin\phi\cos\phi.$$

   What is the direction of the group velocity vector relative to the wave number? Explain.
   Reading Chapter 10 of Cushman may help you to solve this question.



5) <u>Lee waves:</u>
A 10-m/s wind blows over a rugged terrain, and lee waves are generated. If the stratification frequency is equal to 0.03 s^{-1} and if the topography is approximated to a sinusoidal pattern aligned perpendicularly to the wind, with a 25-km wavelength and a height difference from trough to crest of 500 m, calculate the vertical wavelength, the angle made by the wave fronts (surfaces of constant phase) with the horizontal, and the maximum horizontal velocity at the ground. Also, where is this maximum velocity observed (at crests, at troughs, or at the points of maximum slope)?