תורת היחסות הפרטית, פרק Kleppner 12,14

The Lorentz transformations are simplified by introducing

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}.$$

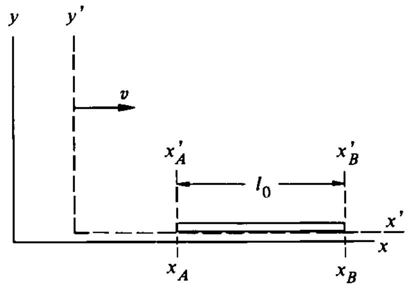
Since $(v/c)^2 \leq 1$, γ is greater than or equal to one. The Lorentz transformations, Eqs. (11.3) and (11.4), then take the form

$$\begin{aligned} x' &= \gamma(x - vt) & x &= \gamma(x' + vt') \\ y' &= y & y &= y' \\ z' &= z & z &= z' \\ t' &= \gamma\left(t - \frac{xv}{c^2}\right) & t &= \gamma\left(t' + \frac{x'v}{c^2}\right). \end{aligned} \qquad 12.1$$

The Twin Paradox

The paradox is as follows: two identical twins, A and B for short, have identical clocks. B sets out on a long space voyage while A remains home. A constantly observes B's clock and sees that it is running slow due to time dilation. Eventually B returns home. Since B's clock has run slow throughout the trip, A concludes that B is younger than A at the end of the journey. But suppose we look at the situation from B's point of view. Since time dilation depends only on relative motion, during the trip B sees A's clock running slow, and when the trip is finished B concludes that A is younger than B. Obviously both twins can't be right. Is either twin really younger?

The Lorentz Contraction



$$x'_B = \gamma(x_B - vt)$$
$$x'_A = \gamma(x_A - vt).$$

Subtracting, we obtain $l_0 = \gamma l$, or

$$l = \frac{l_0}{\gamma} = l_0 \sqrt{1 - \frac{v^2}{c^2}}.$$

l is shorter than l_0: the meter stick is contracted. As $v \to c$, $l \to 0$. This shortening, known as the Lorentz contraction, occurs only along the direction of motion: if the stick lay along the y axis, we would use the transformation $y' = y$ to find $l_0 = l$.

שאלת הכנה 1:

A rod of length l_0 lies in the $x'y'$ plane of its rest system and makes an angle θ_0 with the x' axis. What is the length and orientation of the rod in the lab system x, y in which the rod moves to the right with velocity v?