Page of formulas and values for important constants for the Physics 3A Exam

De Broglie Wavelength: $\lambda = h/p$. Wavenumber: $k = 2\pi/\lambda$. Momentum: $p = mv = \hbar k$.

$\hbar = h/2\pi$. De Broglie frequency: $\nu = E/h$. Kinetic energy: $E = \frac{p^2}{2m} = \frac{1}{2}mv^2$. De Broglie phase velocity: $c = \lambda\nu$.

TISE: $\hat{H}\psi_n = E_n\psi_n$. Hamiltonian: $\hat{H} = \frac{\hat{p}^2}{2m} + V(x) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)$.

Momentum: $\hat{p}_x = -i\hbar\frac{\partial}{\partial x}$. $\hat{\vec{p}} = -i\hbar\nabla$.

TDSE: $i\hbar\frac{\partial\Psi(x,t)}{\partial t} = \hat{H}\Psi(x,t)$.

Born interpretation: $p(x,t) = \Psi^*(x,t)\Psi(x,t) = |\Psi(x,t)|^2$.

Expectation value: $\langle A\rangle = \int_{-\infty}^{\infty}\Psi^*(x,t)\,\hat{A}\Psi(x,t)dx$. For a general probability density:

$\langle A\rangle = \int_{-\infty}^{\infty}p(x,t)\,A(x)dx$. For discrete variable: $\langle A\rangle = \sum_i P_iA_i$.

Commutator: $[\hat{A},\hat{B}] \equiv \hat{A}\hat{B} - \hat{B}\hat{A}$. Standard deviation (uncertainty): $\Delta A = \sqrt{\langle A^2\rangle - \langle A\rangle^2}$.

Uncertainty Relation: $\Delta A\Delta B \geq \frac{1}{2}|\langle[\hat{A},\hat{B}]\rangle|$.

Position-momentum commutator: $[\hat{x},\hat{p}] = i\hbar \rightarrow \Delta x\Delta p \geq \frac{\hbar}{2}$.

TISE for harmonic oscillator: $\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \left(\frac{kx^2}{2}\right)\right)\psi(x) = E_n\psi(x)$

Energy levels of harmonic oscillator: $E_n = \left(n+\frac{1}{2}\right)\sqrt{\frac{k}{m}}\hbar = \hbar\omega\left(n+\frac{1}{2}\right)$. Harmonic oscillator raising and lowering operators: $\hat{a}^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} - \frac{i}{m\omega}\hat{p}\right)$; $\hat{a} = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} + \frac{i}{m\omega}\hat{p}\right)$.

$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$. $\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$.

Dirac notation: $\langle n|m\rangle = \int_{-\infty}^{\infty}\psi_n^*(x)\psi_m(x)dx = \delta_{nm}$. $\langle n|\hat{A}|m\rangle = \int_{-\infty}^{\infty}\psi_n^*(x)\hat{A}\psi_m(x)dx$.

Angular momentum: $\hat{L}_x = -i\hbar\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)$, $\hat{L}_y = -i\hbar\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right)$, $\hat{L}_z = -i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)$.

$$[\hat{L}_x,\hat{L}_y] = i\hbar\hat{L}_z; [\hat{L}_y,\hat{L}_z] = i\hbar\hat{L}_x; [\hat{L}_z,\hat{L}_x] = i\hbar\hat{L}_y;$$

Angular momentum in spherical coordinates: $\hat{L}_x = i\hbar\left(sin(\phi)\frac{\partial}{\partial\theta} + cot(\theta)\,cos(\phi)\frac{\partial}{\partial\phi}\right)$, $\hat{L}_y = i\hbar\left(-\cos(\phi)\frac{\partial}{\partial\theta} + cot(\theta)\,sin(\phi)\frac{\partial}{\partial\phi}\right)$, $\hat{L}_z = -i\hbar\frac{\partial}{\partial\phi}$.

$\hat{\vec{L}}^2 = -\hbar^2 \left(\frac{1}{sin(\theta)}\frac{\partial}{\partial\theta} \left(sin(\theta)\frac{\partial}{\partial\theta}\right) + \frac{1}{sin^2(\theta)}\frac{\partial^2}{\partial\phi^2} \right)$. $\hat{L}_\pm = \hat{L}_x \pm i\hat{L}_y$;

$\hat{L}_z|l, m_l\rangle = \hbar m_l|l, m_l\rangle$; $\hat{L}^2|l, m_l\rangle = \hbar^2 l(l+1)|l, m_l\rangle$;

$\hat{L}_\pm|l, m_l\rangle = \hbar\sqrt{l(l+1) - m_l(m_l \pm 1)}|l, m_l \pm 1\rangle$

Laplacian in spherical coordinates:

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\, sin^2\theta}\frac{\partial^2}{\partial\phi^2}$$

Hydrogen atom: $\psi_{nlm}(\vec{r}) = R_{nl}(r)Y_{lm}(\theta, \phi)$; $\hat{H}\psi_{nlm}(\vec{r}) = -\frac{\mu Z^2 e^4}{2\hbar^2 n^2}\psi_{nlm}(\vec{r})$;$\mu = \frac{m_e}{1+\frac{m_e}{M}}$.

Orbital magnetic moment: $\mu_{orb,z} = -m_l\mu_B$. Magnetic dipole interaction energy: $V_B = -\vec{\mu}\cdot\vec{B}$.

Volume element in cartesian coordinates: $dxdydz$.

Volume element in spherical coordinates: $r^2 sin\theta dr d\theta d\phi$.

Boltzmann distribution: $f_B(\varepsilon) = \frac{1}{Z}e^{-\frac{\varepsilon}{k_B T}}$; $Z = \int e^{-\frac{\varepsilon}{k_B T}}$ the integral is over all possible states

Fermi Dirac distribution: $f_{FD}(\varepsilon) = \frac{1}{1+e^{\frac{\varepsilon-\mu}{k_B T}}}$; Chemical potential: $\mu = \mu(T).\mu(T=0) = \varepsilon_F$.

Reflection and Transmission coefficients: $R = \frac{j_{ref}}{j_{inc}} = \frac{|\psi_{ref}|^2}{|\psi_{inc}|^2}$, $T = \frac{j_{trans}}{j_{inc}} = \frac{k_{trans}|\psi_{trans}|^2}{k_{inc}|\psi_{inc}|^2}$,

$R + T = 1$.

Charge density: $\rho = q|\psi|^2$. Velocity: $\langle\vec{v}\rangle = \frac{\langle\vec{p}\rangle}{m} = \frac{\hbar\langle\vec{k}\rangle}{m}$. Current density: $\vec{j} = nq\langle\vec{v}\rangle = \rho\langle\vec{v}\rangle$

Trigonometric identities:

$\sin^2(\theta) = \frac{1}{2}(1 - \cos(2\theta))$; $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$;$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$;

$e^{i\theta} = \cos(\theta) + isin(\theta)$

Constants:

Electron mass: $9.11 \times 10^{-31}kg$. Proton mass: $1.673 \times 10^{-27}kg$. Electron charge: $e = 1.6 \times 10^{-19}C$. Electron volt: $1eV = 1.6 \times 10^{-19}J$. Planck constant: $h = 6.63 \times 10^{-34}J\cdot s$. $\hbar = 1.054 \times 10^{-34}J\cdot s$. Speed of light in vacuum: $c = 3 \times 10^8 m/s$. Rest energy of electron: $m_e c^2 = 0.511 \times 10^6 eV$. Bohr magneton: $9.274 \times 10^{-24}J\cdot T^{-1}$.Boltzmann constant: $k_B = 1.38 \times 10^{-23}J\cdot K^{-1} = 8.62 \times 10^{-5}eV\cdot K^{-1}$.. Avogadro number: $N_A = 6.022 \times 10^{23}$.

Useful integrals:

$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$, $\int_{-\infty}^{\infty} e^{-ax^2+bx} dx = \sqrt{\pi/a} e^{\frac{b^2}{4a}}$, $\int_0^{\infty} \sqrt{x} e^{-nx} dx = \frac{\sqrt{\pi}}{2n^{3/2}}$,

$\int_0^{\infty} x^n e^{-x} dx = \Gamma(n+1)\ for\ \ n > -1$; If $n \in Integers$ and $n \geq 1$ $\Gamma(n) = n-1!$

$\int_0^{2\pi} \cos^2(x)\, dx = \pi$, $\int \sin^2(x)\, x dx = \frac{x^2}{4} - \frac{1}{8}\cos(2x) - \frac{1}{4} x sin(2x)$.

$\int_{-\infty}^{\infty} \frac{1}{x^2} \sin^2(x)\, dx = \pi$

Fourier transform:

$\mathcal{F}(f(x)) = \tilde{f}(k) = \int_{-\infty}^{\infty} f(x) e^{ikx} dx$ $\mathcal{F}^{-1}\left(\tilde{f}(k)\right) = f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{f}(k) e^{-ikx} dk$

$$\mathcal{F}\left(\frac{df(x)}{dx}\right) = -ik\tilde{f}(k)$$

Dirac's delta function: $\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i(k-k')x} dx = \delta(k-k')$ $\int_{-\infty}^{\infty} f(x)\delta(x-a)dx = f(a)$

Electromagnetic field:

Force due to electric field: $\vec{F} = q\vec{E}$. Lorentz force: $\vec{F} = \frac{q}{c}\ \vec{v} \times \vec{H}$. Energy density in vacuum:

$u = \frac{1}{2}\left(\varepsilon_0\, \vec{E}^2 + \mu_0\, \vec{H}^2\right)$. Ohm's law: $\vec{j} = \underline{\underline{\sigma}} \cdot \vec{E}$, conductivity tensor: $\underline{\underline{\sigma}} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}$.

Drude's isotropic conductivity $\sigma_0 = \frac{ne^2\tau}{m_e}$

Heat capacity:

$$c_V = \left(\frac{\partial u}{\partial T}\right)_V \quad , u = \frac{U}{V} \quad U\ is\ the\ intenal\ energy\ and\ V\ the\ volume$$

Density of states for "electron gas":

3D: $g(\varepsilon) = \frac{3n}{2\varepsilon_F}\sqrt{\frac{\varepsilon}{\varepsilon_F}}$