Question :

Let $\varphi_{nlm}(r)$ denote the properly-normalized energy eigenfunctions of the Hydrogen atom (H-atom) with principal quantum number *n* and angular momentum quantum numbers ℓ and *m*.

Consider an electron in the following state which is a superposition of H-atom eigenstates:

$$\psi(r) = c[\varphi_{100}(r) + 4i\varphi_{210}(r) - 2\sqrt{2}\varphi_{22-1}(r)\ ]$$

(a) Find the normalization constant *C* . Can *C* be complex, real or imaginary?

(b) What is the expectation value of the Hamiltonian for the H-atom in this superposition of states? Calculate your answer in units of eV.

In parts (c) and (d), find the following expectation values:

(c) Find $\langle L^2 \rangle$.

(d) Find $\langle L_z \rangle$.

(e) What is the probability of finding the electron in the $\varphi_{210}(r)$ state?

(f) If the electron is in the $\varphi_{100}(r)$ state, find the radius at which the radial probability density is a maximum. How does this compare with the Bohr radius? What is the significance?

(g) At time $t = 0$, the z-component of the angular momentum $L_z = 0$. Find the wavefunction $\psi(r, t)$ at some later time t, and the probability to measure an energy corresponding to the principal number $n = 2$.