דף נוסחאות - פיסיקה 3

זהויות טריגונומטריות:

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \qquad \sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta \qquad \cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta} \qquad \tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

$$\sin\alpha \pm \sin\beta = 2\sin\frac{\alpha\pm\beta}{2}\cos\frac{\alpha\mp\beta}{2} \qquad \sin\alpha\sin\beta = \frac{1}{2}\left(\cos(\alpha-\beta) - \cos(\alpha+\beta)\right)$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \qquad \cos\alpha\cos\beta = \frac{1}{2}\left(\cos(\alpha-\beta) + \cos(\alpha+\beta)\right)$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} \qquad \sin\alpha\cos\beta = \frac{1}{2}\left(\sin(\alpha-\beta) + \sin(\alpha+\beta)\right)$$

$$\tan\alpha \pm \tan\beta = \frac{\sin(\alpha\pm\beta)}{\cos\alpha\cos\beta} \qquad \tan^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{1+\cos\alpha}$$

$$\sin^2\frac{\alpha}{2} = \frac{1}{2}(1-\cos\alpha) \qquad \cos^2\frac{\alpha}{2} = \frac{1}{2}(1+\cos\alpha)$$

$$\sin^2\alpha + \cos^2\alpha = 1$$

זהויות אויילר:

$$\cos x = \frac{e^{ix}+e^{-ix}}{2} \qquad \sin x = \frac{e^{ix}-e^{-ix}}{2i}$$
$$e^{ix} = \cos x + i\sin x$$

פורייה:

טור ממשי:

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n\cos\left(\frac{2\pi n}{b-a}x\right) + b_n\sin\left(\frac{2\pi n}{b-a}x\right)\right]$$
$$a_n = \frac{2}{b-a}\int_a^b f(x)\cos\left(\frac{2\pi n}{b-a}x\right)dx, \qquad b_n = \frac{2}{b-a}\int_a^b f(x)\sin\left(\frac{2\pi n}{b-a}x\right)dx$$

טור מרוכב:

$$f\left(x\right)\sim\sum_{n=-\infty}^{\infty}c_n\exp\left(i\frac{2\pi n}{b-a}x\right)$$

$$c_n=\frac{2}{b-a}\int\limits_a^b f\left(x\right)\exp\left(-i\frac{2\pi n}{b-a}x\right)dx$$

התמרה:

$$\hat{f}\left(\omega\right)=\frac{1}{2\pi}\int\limits_{-\infty}^{\infty}f\left(x\right)e^{-i\omega x}dx$$

$$f\left(x\right)=\int\limits_{-\infty}^{\infty}\hat{f}\left(\omega\right)e^{i\omega x}d\omega$$

$$f\left(x\right)=\int\limits_0^{\infty}A\left(\omega\right)\cos\left(\omega x\right)d\omega+\int\limits_0^{\infty}B\left(\omega\right)\sin\left(\omega x\right)d\omega$$

$$A\left(\omega\right)=\frac{1}{\pi}\int\limits_{-\infty}^{\infty}f\left(x\right)\cos\left(\omega x\right)dx,\qquad B\left(\omega\right)=\frac{1}{\pi}\int\limits_{-\infty}^{\infty}f\left(x\right)\sin\left(\omega x\right)dx$$

התמרות נפוצות:

$$F\left[e^{-x^2}\right]=\frac{1}{2\sqrt{\pi}}e^{-\frac{\omega^2}{4}}$$

$$F\left[e^{-|x|}\right]=\frac{1}{\pi\left(\omega^2+1\right)}$$

$$F\left[f^{(n)}\left(x\right)\right]=\left(i\omega\right)^n\hat{f}\left(\omega\right)$$

$$F\left[\chi_{[a,b]}\right]=\frac{e^{-ia\omega}-e^{-ib\omega}}{2\pi i\omega}$$

$$F\left[x^nf\left(x\right)\right]=i^n\frac{d^n}{d\omega^n}\hat{f}\left(\omega\right)$$

$$F\left[f\left(ax+b\right)\right]=\frac{1}{|a|}e^{i\frac{\omega b}{a}}\hat{f}\left(\frac{\omega}{a}\right)$$

אינטגרלים מידיים:

$$\int\frac{dx}{x^2-a^2}=\frac{1}{2a}\ln\frac{x-a}{x+a}\qquad\int\frac{dx}{x^2+a^2}=\frac{1}{a}\arctan\frac{x}{a}$$

$$\int\frac{dx}{\sqrt{x^2\pm a^2}}=\ln\left(x+\sqrt{x^2\pm a^2}\right)\qquad\int\frac{dx}{\sqrt{a^2-x^2}}=\arcsin\frac{x}{|a|}$$

$$\int\tan x\,dx=-\ln\cos x\qquad\int\cot x\,dx=\ln\sin x$$

$$\int\tanh x\,dx=\ln\cosh x\qquad\int\coth x\,dx=\ln\sinh x$$

$$\int\frac{dx}{\sin x}=\ln\tan\frac{x}{2}\qquad\int\frac{dx}{\cos x}=\ln\left(\tan x+\frac{1}{\cos x}\right)$$

פיתוחי טיילור ידועים:

$$(1+x)^p = 1 + p\,x + \frac{p(p-1)}{2!}\,x^2 + \ldots + \frac{p(p-1)(p-2)\ldots(p-n+1)}{n!}\,x^n + \ldots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \ldots + (-1)^{n+1}\,\frac{x^n}{n} + \ldots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!} + \ldots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots + (-1)^n\,\frac{x^{2n+1}}{(2n+1)!} + \ldots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots + (-1)^n\,\frac{x^{2n}}{(2n)!} + \ldots$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots + \frac{x^{2n+1}}{(2n+1)!} + \ldots$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots + \frac{x^{2n}}{(2n)!} + \ldots$$

סכום סדרה הנדסית:

$$S_n = \sum_{i=1}^{n} q^{i-1} = \frac{1-q^n}{1-q} \qquad S = \sum_{i=0}^{\infty} q^i = \frac{1}{1-q}$$

זהויות ואופרטורים וקטוריים:

$$\nabla f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z} = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$$

$$\nabla \cdot \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} = \frac{1}{r^2}\frac{\partial\left(r^2 F_r\right)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial\left(\sin\theta F_\theta\right)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial\left(F_\phi\right)}{\partial\phi}$$

$$\nabla \times \vec{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{x} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{y} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{z} =$$

$$= \frac{1}{r\sin\theta}\left(\frac{\partial\left(\sin\theta F_\phi\right)}{\partial\theta} - \frac{\partial F_\theta}{\partial\phi}\right)\hat{r} + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial F_r}{\partial\phi} - \frac{\partial\left(rF_\phi\right)}{\partial r}\right)\hat{\theta} + \frac{1}{r}\left(\frac{\partial\left(rF_\theta\right)}{\partial r} - \frac{\partial F_r}{\partial\theta}\right)\hat{\phi}$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$$

$$A \times (B \times C) = (C \times B) \times A = B\,(A \cdot C) - C\,(A \cdot B)$$

$$\nabla\,(A \cdot B) = A \times (\nabla \times B) + (A \cdot \nabla)\,B + B \times (\nabla \times A) + (B \cdot \nabla)\,A$$

$$\nabla \cdot (A \times B) = B \cdot \nabla \times A - A \cdot \nabla \times B$$

$$\nabla \times (A \times B) = (B \cdot \nabla)\,A + A\,(\nabla \cdot B) - (A \cdot \nabla)\,B - B\,(\nabla \cdot A)$$

משוואות חשמל:

מקסוול:

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$
$$\nabla \cdot \vec{D} = 4\pi\rho$$
$$\vec{D} = \varepsilon \vec{E}$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \frac{4\pi}{c}\vec{J}$$
$$\nabla \cdot \vec{B} = 0$$
$$\vec{H} = \frac{\vec{B}}{\mu}$$

חוק אוהם:

$$\vec{J} = \sigma \vec{E}$$

מתחים של רכיבים חשמליים:

$$V_R = IR \qquad V_C = \frac{Q}{C} \qquad V_L = L\dot{I}$$

חוקי קירכהוף:

$$\sum I_{in} = \sum I_{out} \qquad \sum V = 0$$

משוואות מתרמודינמיקה:

משוואת מצב של גז אידיאלי:

$$PV = NRT$$

משוואת פואסון לתהליכים אדיאבטיים:

$$\frac{P}{\rho^\gamma} = \text{const}$$

משוואת הרציפות:

$$Sv = \text{const}$$