Gravity 1 - Tutorial 3

Special Relativity

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Contents

1 Time and Distance

An inertial observer in frame x^μ measures simultaneously ($\Delta t = 0s$) a length of $\Delta x = 3m$. Another observer $x^{\mu'}$ is traveling at constant speed v along the x - direction. In his frame he measures between the same two events the time interval $\Delta t' = 10^{-8}s$. The speed of light is $c = 3 \cdot 10^8 \frac{m}{s}$.

Find $\Delta x'$ in three ways: 1. using Lorentz transformations. 2. using the invariant line element (quadrance). 3. using geometry.

1.1 Solving with Lorentz Transformations

We solve in two steps. First step: find γ by transforming the t component. Second step: find $\Delta x'$ by transforming the x component.

Find γ

$$c\Delta t' = \gamma \left(c\Delta t - \beta \Delta x\right) \tag{1}$$

plug in the data

$$1 = -\gamma\beta \tag{2}$$

Notice that β is negative (so $v < 0$).

$$1 = \gamma^2\beta^2 = \frac{\beta^2}{1-\beta^2} \tag{3}$$

$$\beta^2 = \frac{1}{2} \tag{4}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\frac{1}{2}}} = \sqrt{2} \tag{5}$$

Find $\Delta x'$

$$\Delta x' = \gamma \left(\Delta x - \beta \Delta t\right) \tag{6}$$

$$\Delta x' = \gamma \Delta x = 3\sqrt{2} = \sqrt{18}\,[m] \tag{7}$$

1.2 Solving with Invariant Line Element

We solve in two steps. First step: Compute $(\Delta s)^2$. Second step: Find $\Delta x'$ by demanding $(\Delta s')^2 = (\Delta s)^2$.

$$(\Delta s)^2 = -(c\Delta t)^2 + (\Delta x)^2 = 9\left[m^2\right] \tag{8}$$

$$(\Delta s)^2 = (\Delta s')^2 = -(c\Delta t')^2 + (\Delta x')^2 \tag{9}$$

$$(\Delta x')^2 = (\Delta s)^2 + (c\Delta t')^2 = 9 + 3^2 = 18\left[m^2\right] \tag{10}$$

$$\Delta x' = \sqrt{18}\,[m] \tag{11}$$

1.3 Solving with Geometry

See Figure 1.

Consider the tx coordinate axes of the first inertial observer (in black). Let us put the first event at the origin $P_1 = (0,0)$, hence the second event is at $P_2 = (0,3)$. The two observers are together at P_1 and start their clocks at the same moment. The rest frame of the second inertial observer with $\beta = \frac{v}{c} = \frac{x}{ct}$ has coordinate axes $t'x'$ (in red). We want to find $\Delta x'$, which is the x' coordinate of event P_2. It is the distance of the point A from the origin. Therefore, what we need to compute is $\Delta x' = \sqrt{q\,(P_1 A)}$.

We will solve in two ways, analogue to the two previous algebraic solutions.

1.3.1 Find Point A

We will use the following line equations:

The equation of the ct' axis is

$$ct = \frac{1}{\beta}x \tag{12}$$

The equation of the dotted line parallel to the ct' axis through P_2 is

$$ct = \frac{1}{\beta}(x-3) \tag{13}$$

The equation of the x' axis is

$$ct = \beta x \tag{14}$$

The equation of the dotted line parallel to the x' axis through P_2 is

$$ct = \beta(x-3) \tag{15}$$

As in section 1.1, first we find β and γ, from the data given on point B. Point B is the intersection of lines (12) and (15). We solve for x_B

$$\frac{1}{\beta}x_B = \beta x_B - 3\beta \tag{16}$$

$$x_B = \frac{-3\beta^2}{1-\beta^2} \tag{17}$$

Plug in (12) to find ct_B

$$ct_B = \frac{-3\beta}{1-\beta^2} \tag{18}$$

It is given that $c\Delta t' = 3$, thus the quadrance of B is $q\left(B\right) = -\left(ct_B\right)^2 + x_B^2 = -9$.

$$-\frac{9\beta^2}{\left(1-\beta^2\right)^2} + \frac{9\beta^4}{\left(1-\beta^2\right)^2} = -9 \tag{19}$$

solve for β

$$\beta^2 = \frac{1}{2}, \quad \beta = -\frac{1}{\sqrt{2}} \quad \gamma^2 = 2 \tag{20}$$

Now we find point A. It is the intersection of the line parallel to ct' axis through P_2 (13) and the x' axis (14). We solve for x_A

$$\beta x_A = \frac{1}{\beta}\left(x_A - 3\right) \tag{21}$$

$$x_A = \frac{3}{1-\beta^2} = 6 \tag{22}$$

Plug in (14)

$$t_A = -\frac{6}{\sqrt{2}} \tag{23}$$

Now we can calculate the quadrance of A

$$q\left(A\right) = -\left(c\Delta t_A\right)^2 + x_A^2 = -\frac{36}{2} + 36 = 18 \tag{24}$$

$$\Delta x' = \sqrt{q\left(A\right)} = \sqrt{18} \tag{25}$$

1.3.2 Use Pythagoras Theorem

The shorter way, analogue to section 1.2, is not finding β but using Pythagoras theorem for the triangle $\Delta P_1 P_2 A$. The sides $P_1 A$ and $P_2 A$ are orthogonal because they are in the directions of x' and t' axes, which are given by a rotation

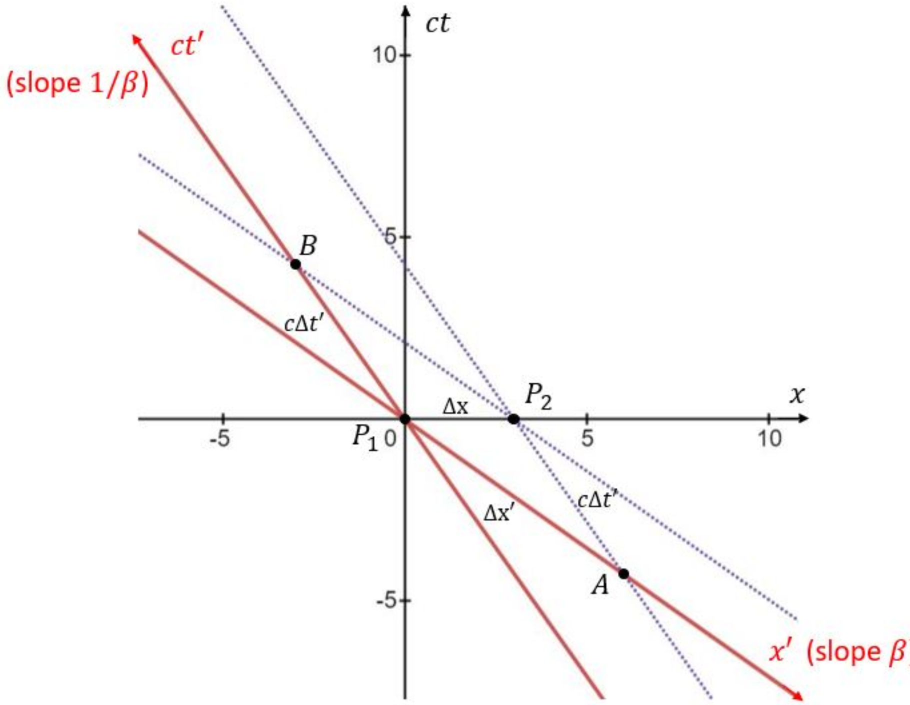

Figure 1: Relative Moving Frames

(Lorentz) of t and x axes, thus the orthogonality is preserved. Pythagoras theorem is

$$q\left(P_1A\right)+q\left(P_2A\right)=q\left(P_1P_2\right) \tag{26}$$

Because the tx and $t'x'$ coordinates are related by an isometry we can use the same quadrance formula for both, so $q\left(P_1B\right)=-\left(c\Delta t'\right)^2$, $q\left(P_1A\right)=\Delta x'^2$ and $q\left(P_1P_2\right)=\Delta x^2$. $q\left(P_2A\right)=q\left(P_1B\right)$ since BP_2AP_1 is parallelogram (actually a rectangle).

$$\Delta x'^2+\left(-\left(c\Delta t'\right)^2\right)=\Delta x^2 \tag{27}$$

$$\Delta x'^2=\left(c\Delta t'\right)^2+\Delta x^2=3^2+3^2=18 \tag{28}$$

2 Velocity and Acceleration

2.1 Boost Along the x - Direction

In an inertial frame x^μ a particle is moving with a 3-velocity $u^i = \left(u^1, u^2, u^3\right)$ (Cartesian coordinates). Another inertial frame $x^{\mu'}$ is moving with relative constant velocity v along the x - direction. The 3-velocities in the $x^{\mu'}$ frame are

3-velocity transformation for Boost along x-direction

$$u^{1'} = \frac{u^1 - v}{1 - \frac{u^1 v}{c^2}} \tag{29}$$

$$u^{2'} = \frac{u^2}{\gamma\left(1 - \frac{u^1 v}{c^2}\right)} \tag{30}$$

$$u^{3'} = \frac{u^3}{\gamma\left(1 - \frac{u^1 v}{c^2}\right)} \tag{31}$$

Lets look at two special cases.

First case - a light moving in the x - direction, so $u^i = (c, 0, 0)$. Plug into (29)(30)(31) we find

$$u^{1'} = \frac{c - v}{1 - \frac{cv}{c^2}} = \frac{c\,(c - v)}{c - v} = c \tag{32}$$

and $u^{2'} = u^{3'} = 0$. In the moving frame the light has the same speed and same direction.

Second case - a light moving in the y - direction, so $u^i = (0, c, 0)$. Plug into (29)(30)(31) we find

$$u^{1'} = -v \tag{33}$$

$$u^{2'} = \frac{c}{\gamma} \tag{34}$$

and $u^{3'} = 0$. (33) is like a Galilean transformation. The light has no velocity in the x direction in the original frame. In the frame moving with velocity v in the x direction it has velocity v in this opposite direction. A Galilean transformation would yield $u^{2'} = c$, yet (34) corrects this. The velocity in the y direction is reduced by the Lorentz factor, such that the **total speed** of light would not change.

$$\begin{aligned} |\boldsymbol{u'}|^2 &= \left(u^{1'}\right)^2 + \left(u^{2'}\right)^2 + \left(u^{3'}\right)^2 = v^2 + \frac{c^2}{\gamma^2} \\ &= c^2\left(\left(\frac{v}{c}\right)^2 + \gamma^{-2}\right) = c^2\left(\beta^2 + 1 - \beta^2\right) = c^2 \end{aligned} \tag{35}$$

2.2 Light Ray at Some Angle

In a lab rest frame, a light ray is traveling in the xy plane in a direction of angle θ, counter clock-wise with respect to the x axis. An observer $x^{\mu'}$ is traveling at constant speed v along the positive x - direction.

1. Show that in the observer's frame the light ray has speed c.
2. What is the angle θ' between the light ray and the x' axis?

2.2.1 Calculating the speed of light

Let us use units with $c = 1$.

The velocity in the lab frame is

$$u^i = (cos\theta, sin\theta, 0) \tag{36}$$

By (29)(30)(31), the velocity in the observer's frame is

$$u^{i'} = \left(\frac{cos\theta - v}{1 - vcos\theta}, \frac{sin\theta}{\gamma\left(1 - vcos\theta\right)}, 0\right) \tag{37}$$

$$\begin{aligned}
|\boldsymbol{u'}|^2 = \left(u^{1'}\right)^2 + \left(u^{2'}\right)^2 + \left(u^{3'}\right)^2 = \left(\frac{cos\theta - v}{1 - vcos\theta}\right)^2 + \left(\frac{sin\theta}{\gamma\left(1 - vcos\theta\right)}\right)^2 \\
= &\frac{1}{\left(1 - vcos\theta\right)^2}\left(cos^2\theta - 2vcos\theta + v^2 + \left(1 - v^2\right)sin^2\theta\right) \\
= &\frac{1}{\left(1 - vcos\theta\right)^2}\left(cos^2\theta + sin^2\theta - 2vcos\theta + v^2\left(1 - sin^2\theta\right)\right) \\
= &\frac{1}{\left(1 - vcos\theta\right)^2}\left(1 - 2vcos\theta + v^2cos^2\theta\right) = \frac{\left(1 - vcos\theta\right)^2}{\left(1 - vcos\theta\right)^2} = 1 = c^2
\end{aligned} \tag{38}$$

2.2.2 Calculating the angle

$$tan\theta' = \frac{u^{2'}}{u^{1'}} = \frac{sin\theta}{\gamma\left(cos\theta - v\right)} = \frac{\sqrt{1 - v^2}sin\theta}{cos\theta - v} \tag{39}$$

2.3 The Acceleration 4-vector

The *acceleration 4-vector* is defined as

$$a^\mu := \frac{du^\mu}{d\tau} \tag{40}$$

Recall that since we work with arc-length parameter τ, a particle has a constant 4-speed $u^\mu u_\mu$. The acceleration of a particle with constant speed is perpendicular to its velocity.

$$a^\mu u_\mu = 0$$

Proof: If $u^2 = u \cdot u = const$, then $\frac{d}{d\tau}\left(u^2\right) = 2u \cdot \frac{du}{d\tau} = 2u \cdot a = 0$. The classic example for this is a particle moving along a circle with constant speed. It has only a radial acceleration, perpendicular to the velocity which is tangent to the circle.

Exercise:

1. Express the 4-acceleration a^μ with the 3-velocity $\boldsymbol{v} = \frac{d\boldsymbol{x}}{dt}$ and 3-acceleration $\boldsymbol{a} = \frac{d\boldsymbol{v}}{dt}$.
2. Check that $a^\mu u_\mu = 0$.
3. Write down the 4-acceleration in the rest frame of the accelerating particle.

Solution:

1.

$$a^\mu = \frac{du^\mu}{d\tau} = \frac{du^\mu}{dt}\frac{dt}{d\tau} = \gamma\frac{du^\mu}{dt} = \gamma\left(\frac{d}{dt}\gamma, \frac{d}{dt}(\gamma\boldsymbol{v})\right) = \left(\gamma\frac{d\gamma}{dt}, \gamma\frac{d\gamma}{dt}\boldsymbol{v} + \gamma^2\frac{d\boldsymbol{v}}{dt}\right) \tag{41}$$

where we used

$$u^\mu = \left(\frac{dt}{d\tau}, \frac{d\boldsymbol{x}}{d\tau}\right) = (\gamma, \gamma\boldsymbol{v}) \tag{42}$$

Calculate $\frac{d\gamma}{dt}$

$$\frac{d\gamma}{dt} = \frac{d}{dt}\left(1-\boldsymbol{v}^2\right)^{-\frac{1}{2}} = \left(1-\boldsymbol{v}^2\right)^{-\frac{3}{2}}\boldsymbol{v}\cdot\frac{d\boldsymbol{v}}{dt} = \gamma^3\boldsymbol{v}\cdot\boldsymbol{a} \tag{43}$$

Plug into (41) we find

$$\boxed{a^\mu = \left(\gamma^4\boldsymbol{v}\cdot\boldsymbol{a}\ ,\ \gamma^4(\boldsymbol{v}\cdot\boldsymbol{a})\boldsymbol{v} + \gamma^2\boldsymbol{a}\right)} \tag{44}$$

4-acceleration from 3-velocity

2.

$$\begin{aligned} a^\mu u_\mu &= -\gamma^4(\boldsymbol{v}\cdot\boldsymbol{a})\gamma + \left(\gamma^4(\boldsymbol{v}\cdot\boldsymbol{a})\boldsymbol{v} + \gamma^2\boldsymbol{a}\right)\cdot(\gamma\boldsymbol{v}) \\ &= -\gamma^5(\boldsymbol{v}\cdot\boldsymbol{a})\left(1-\boldsymbol{v}^2-\gamma^{-2}\right) = -\gamma^5(\boldsymbol{v}\cdot\boldsymbol{a})\left(\gamma^{-2}-\gamma^{-2}\right) = 0 \end{aligned} \tag{45}$$

3. In the rest frame $\boldsymbol{v} = 0$ and $\gamma = 1$. (44) becomes

$$a^{\mu} = (0, \boldsymbol{a}) \tag{46}$$

Acceleration in the rest frame

3 Motion with Uniform Acceleration

A particle is moving along the x-axis. It is uniformly accelerated in the sense that the acceleration measured in its instantaneous rest frame is always g, a constant. Find $x(\tau)$ and $t(\tau)$ as functions of the proper time assuming the particle passes through x_0 at time $t = 0$ with zero velocity. Draw the worldline of the particle on a spacetime diagram.

The four acceleration of accelerated particle is

$$a^{\mu} = \left(\gamma^4 \boldsymbol{v} \cdot \boldsymbol{a}, \gamma^4 (\boldsymbol{v} \cdot \boldsymbol{a}) \boldsymbol{v} + \gamma^2 \boldsymbol{a}\right) \tag{47}$$

In the accelerated particle's rest frame (τ, ξ), $\boldsymbol{v} = \boldsymbol{0}$ and $a = g$

$$a^{\mu} = (0, g) \tag{48}$$

$$a^{\tau} = 0 \qquad a^{\xi} = g \tag{49}$$

$$u^{\mu} = (1, 0) \tag{50}$$

$$u^{\tau} = 1 \qquad u^{\xi} = 0 \tag{51}$$

At each moment we make a Lorentz transformation (of different velocity) to inertial observer frame (t, x) for the 4-vectors u^{μ} and a^{μ}

$$u^t = \gamma u^{\tau} - \beta\gamma u^{\xi} = \gamma \tag{52}$$

$$u^x = \gamma u^{\xi} - \beta\gamma u^{\tau} = -\beta\gamma \tag{53}$$

$$a^t = \gamma a^{\tau} - \beta\gamma a^{\xi} = -\beta\gamma g \tag{54}$$

$$a^x = \gamma a^{\xi} - \beta\gamma a^{\tau} = \gamma g \tag{55}$$

Plug (53) into (54)

$$a^t = \frac{du^t}{d\tau} = g u^x \tag{56}$$

Plug (52) into (55)

$$a^x = \frac{du^x}{d\tau} = g u^t \tag{57}$$

Differentiate (57) and substitute (56)

$$\frac{d^2u^x}{d\tau^2} = g\frac{du^t}{d\tau} = g^2u^x \tag{58}$$

Solve with initial condition $u^x(0) = 0$ (we set $t(\tau = 0) = 0$)

$$u^x(\tau) = sinh(g\tau) \tag{59}$$

$$u^t(\tau) = cosh(g\tau) \tag{60}$$

Integrate with initial condition $t(0) = 0$, $x(\tau) = x_0$.

$$x(\tau) = x_0 + \frac{1}{g}(cosh(g\tau) - 1) \tag{61}$$

$$t(\tau) = \frac{1}{g}sinh(g\tau) \tag{62}$$

This is a hyperbola. The algebraic form of the curve is

$$t^2 = (x - x_0)^2 + \frac{2}{g}(x - x_0) \tag{63}$$

4 Collisions

4.1 Particles Annihilation/Creation

4.1.1 Annihilation

Two particles with equal masses $m_1 = m_2 = m$ collide head-on. Both have the same speed before the collision $v = \frac{3}{5}$. After the collision there is a particle with mass M at rest. Find M. Is it more/less/equal the sum of the two masses?

Reminder you, the four momentum is

$$p^\mu = (E, \boldsymbol{p}) = (\gamma m, \gamma m\boldsymbol{v}) \tag{64}$$

Solution:
We choose the x axis as the line of the collision. The 4-momenta of the particles

before the collision are

$$p_1^\mu = (\gamma m, \gamma m v, 0, 0)$$
$$p_2^\mu = (\gamma m, -\gamma m v, 0, 0) \tag{65}$$

The total 4-momentum before the collision is

$$p_i^\mu = p_1^\mu + p_2^\mu = (2\gamma m, 0, 0, 0) \tag{66}$$

The total 4-momentum after the collision is

$$p_f^\mu = (M, 0, 0, 0) \tag{67}$$

since $\boldsymbol{v}_f = \mathbf{0}$ and $\gamma_f = 1$.

4-momentum conservation is

$$p_i^\mu = p_f^\mu \tag{68}$$

The energy conservation $p_i^0 = p_f^0$ is

$$\boxed{M = 2m\gamma > 2m} \tag{69}$$

The final mass M is the total energy of the system. It contains both the total mass of the constituent particles and their total kinetic energy.

4.1.2 Creation

A particle at rest with mass M decays (splits) into two particles with equal mass m. Find v.

Due to momentum conservation they have the same speed v. As before, (69)

$$M = 2m\gamma \tag{70}$$

$\Rightarrow$

$$\gamma^{-2} = 1 - v^2 = \left(\frac{2m}{M}\right)^2 \tag{71}$$

$\Rightarrow$

$$v = \sqrt{1 - \left(\frac{2m}{M}\right)^2} \tag{72}$$

There is a solution only for $2m < M$.

4.2 Scattering Of Protons

A proton with energy E collides with another proton at rest. The outcome is four protons. What is the minimal energy E (as a function of the mass of a proton m), for that scattering process to occur?

We will use the fact that $p_\mu p^\mu$ is both invariant under change of reference frame, and conserved with time.

Before the collision, we write the total 4-momentum in the lab frame

$$p^\mu_{lab,i} = (E + m, \boldsymbol{P}) \tag{73}$$

where E and $\boldsymbol{P}$ are the energy and momentum of the moving proton, with $E^2 = m^2 + \boldsymbol{P}^2$. The proton at rest has only a rest energy m.

After the collision, we use the center of mass frame. The sum of the momenta of the four protons is zero in center of mass frame (just as for the two protons before the collision). Still, each proton can have some velocity $\boldsymbol{v}$ relative to the center of mass. We look for minimal energy condition, so we want no energy waste on new kinetic energy for the new protons (except for the center of mass to have the same momentum as before the scattering). It means that in the c.o.m frame we take $\boldsymbol{v} = \boldsymbol{0}$ for all the four protons. Therefore $\gamma = 1$, and each proton contributes to the energy in the c.o.m frame an amount of $E = \gamma m = m$. The final total 4-momentum in the c.o.m frame is

$$p^\mu_{com,f} = (4m, \boldsymbol{0}) \tag{74}$$

Now, since $p_\mu p^\mu$ is both invariant under change of reference frame, and conserved with time, we can compare

$$(p_{lab,i})^\mu (p_{lab,i})_\mu = (p_{com,f})^\mu (p_{com,f})_\mu \tag{75}$$

$\Rightarrow$

$$-(E + m)^2 + \boldsymbol{P}^2 = -(4m)^2 \tag{76}$$

plug $\boldsymbol{P}^2 = E^2 - m^2$

$$-\cancel{E^2} - 2mE - m^2 + \cancel{E^2} - m^2 = -16m^2 \tag{77}$$

$\Rightarrow$

$$E = 7m \tag{78}$$

5 Transformation of the Electromagnetic Field

The electromagnetic field tensor $F^{\mu\nu}$, also called the *field strength* tensor of the electromagnetic field, in an inertial frame is

$$F^{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{pmatrix} \tag{79}$$

It is an *antisymmetric* tensor

$$F^{\mu\nu} = -F^{\nu\mu} \tag{80}$$

What is the electric field $E^{i'}$ in another inertial frame, moving with relative velocity v along the x axis?

To find out, we transform the tensor $F^{\mu\nu}$ and look at the desired components. A $\binom{2}{0}$-tensor transformation rule is

$$F^{\mu'\nu'} = \Lambda^{\mu'}{}_{\rho}\Lambda^{\nu'}{}_{\sigma}F^{\rho\sigma} \tag{81}$$

where

$$\Lambda^{\mu'}{}_{\nu} = \begin{pmatrix} \gamma & -\gamma v & 0 & 0 \\ -\gamma v & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{82}$$

We work with units $c = 1$, so $\beta = v$. When making index calculation we notice which components vanish, so not to write the whole sum explicitly.

The component parallel to v transforms

$$
\begin{aligned}
E'_x = F^{0'1'} = \Lambda^{0'}{}_{\rho}\Lambda^{1'}{}_{\sigma}F^{\rho\sigma} \\
=&\Lambda^{0'}{}_{0}\Lambda^{1'}{}_{1}F^{01} + \Lambda^{0'}{}_{1}\Lambda^{1'}{}_{0}F^{10} \\
=&\gamma^2 E_x + \gamma^2 v^2 \left(-E_x\right) = E_x \gamma^2 \left(1 - v^2\right) = E_x
\end{aligned} \tag{83}
$$

The y - component transforms

$$
\begin{aligned}
E'_y = F^{0'2'} = \Lambda^{0'}{}_{\rho}\Lambda^{2'}{}_{\sigma}F^{\rho\sigma} \\
=&\Lambda^{0'}{}_{0}\Lambda^{2'}{}_{2}F^{02} + \Lambda^{0'}{}_{1}\Lambda^{2'}{}_{2}F^{12} \\
=&\gamma E_y - \gamma v B_z = \gamma \left(E_y + v_z B_x - v_x B_z\right) \\
=&\gamma \left(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}\right)_y
\end{aligned} \tag{84}
$$

The z - component transforms

$$
\begin{aligned}
E'_z = F^{0'3'} = \Lambda^{0'}{}_{\rho}\Lambda^{3'}{}_{\sigma}F^{\rho\sigma} \\
=&\Lambda^{0'}{}_{0}\Lambda^{3'}{}_{3}F^{03} + \Lambda^{0'}{}_{1}\Lambda^{3'}{}_{3}F^{13} \\
=&\gamma E_z - \gamma v \left(-B_y\right) = \gamma \left(E_z + v_x B_y - v_y B_x\right) \\
=&\gamma \left(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}\right)_z
\end{aligned} \tag{85}
$$

where we used that

$$
v^i = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix} \tag{86}
$$

and

$$
\left(\boldsymbol{v} \times \boldsymbol{B}\right)^i = \begin{pmatrix} v_y B_z - v_z B_y \\ v_z B_x - v_x B_y \\ v_x B_y - v_y B_x \end{pmatrix} = \begin{pmatrix} 0 \\ -v B_y \\ v B_y \end{pmatrix} \tag{87}
$$

We conclude, in general that

$$
\boldsymbol{E}'_{\shortparallel} = \boldsymbol{E}_{\shortparallel} \tag{88}
$$

$$
\boldsymbol{E}'_{\perp} = \gamma \left(\boldsymbol{E}_{\perp} + \boldsymbol{v} \times \boldsymbol{B}\right) \tag{89}
$$

Lorentz transformation of the electric field

where $\shortparallel$ and $\perp$ mean parallel and perpendicular to $\boldsymbol{v}$.