Prof. E. Palti January 18, 2023

Particles 1 -Final Project

(Due: February 15th, 2023)

WZ Scattering Unitarity and the Higgs Boson

(This project is working through section 29.2 in Schwartz.)

Note that some questions are marked with an astrix - these are worth a few bonus marks (around 1-1.5% of total course mark each).

We would like to study the scattering of W bosons and Z bosons in the Standard Model. We would like to give them a longitudinal polarization (denoted with L subscript) as external states, and consider the scattering

$$W_L^+(p_1)\,Z_L(p_2) \rightarrow W_L^+(p_3)\,Z_L(p_4)\ . \tag{1}$$

Question 1: Why do the W and Z bosons have a longitudinal polarization? How many degrees of freedom do they posses overall?

Let us take the following external polarization vectors

$$\begin{aligned}\epsilon_1^\mu &= \frac{1}{m_W}p_1^\mu + \frac{2m_W}{t-2m_W^2}p_3^\mu\ ,\quad \epsilon_2^\mu = \frac{1}{m_Z}p_2^\mu + \frac{2m_Z}{t-2m_Z^2}p_4^\mu\ ,\\ \epsilon_3^\mu &= \frac{1}{m_W}p_3^\mu + \frac{2m_W}{t-2m_W^2}p_1^\mu\ ,\quad \epsilon_4^\mu = \frac{1}{m_Z}p_4^\mu + \frac{2m_Z}{t-2m_Z^2}p_2^\mu\ .\end{aligned} \tag{2}$$

We use here the Mandelstam variables

$$s = (p_1+p_2)^2 = k^2\ ,\ \ t = (p_1-p_3)^2\ \ ,\ \ u = (p_1-p_4)^2\ \ , \tag{3}$$

where k^μ is the exchange momentum. Note that the polarization vectors are not normalised to unit norm, but this will not be important for the calculation.

Question 2: What is the constraint on the polarization vectors that ensures they are purely longitudinally polarized? Show that this constraint is indeed satisfied.

There are three tree-level Feynman diagrams which contribute to the scattering process in the Standard Model without the Higgs boson. They are:

1. S-channel exchange involving a W^+ boson.

2. U-channel exchange involving a W^+ boson.
3. the 4-point W^+W^+ZZ interaction vertes.

Question 3: Draw the appropriate Feynman diagrams for the three processes.

Let us consider in detail the S-channel exchange. The matrix element takes the form

$$\begin{aligned}\mathrm{S-channel}\ :\ i\mathcal{M}_s \quad &= \quad (ie\cot\theta_w)^2\, \epsilon_1^\mu \epsilon_2^\nu\, (\epsilon_3^\alpha)^* \left(\epsilon_4^\beta\right)^* \frac{i}{s-m_W^2}\left(-\eta^{\lambda\kappa}+\frac{k^\lambda k^\kappa}{m_W^2}\right) \\ &\times \left[-\eta_{\mu\nu}\left(p_2-p_1\right)_\lambda+\eta_{\nu\lambda}\left(p_2+k\right)_\mu-\eta_{\lambda\mu}\left(k+p_1\right)_\nu\right] \\ &\times \left[-\eta_{\alpha\beta}\left(p_4-p_3\right)_\kappa+\eta_{\beta\kappa}\left(p_4+k\right)_\alpha-\eta_{\kappa\alpha}\left(k+p_3\right)_\beta\right] . \end{aligned} \tag{4}$$

Question 4: Using the Feynman rules show that the matrix element (4) is as given. Explain, in as much detail as possible, each term in the matrix element, in particular stating which terms in the Standard Model Lagrangian they are associated with.

Using the polarization vectors (2), the s-channel matrix element gives

$$\mathcal{M}_s = \frac{e^2\cot^2\theta_w}{4m_W^2 m_Z^2}\left[2su+s^2-2m_W^2\frac{3su+u^2}{s+u}+2m_Z^2\frac{s^2-3su-2u^2}{s+u}-\frac{m_Z^4}{m_W^2}s+\mathcal{O}(1)\right] . \tag{5}$$

Question 5: Show that (5) follows from (4).

The u-channel diagram gives the contribution

$$\mathcal{M}_u = \frac{e^2\cot^2\theta_w}{4m_W^2 m_Z^2}\left[2su+u^2-2m_W^2\frac{3su+s^2}{s+u}+2m_Z^2\frac{u^2-3su-2s^2}{s+u}-\frac{m_Z^4}{m_W^2}u+\mathcal{O}(1)\right] . \tag{6}$$

Question 6* (Bonus Marks): Show the result (6). (Hint: you may look up crossing symmetry.)

The 4-point interaction amplitude gives

$$\mathcal{M}_4 = \frac{e^2\cot^2\theta_w}{4m_W^2 m_Z^2}\left[-s^2-4su-u^2+2\left(m_W^2+m_Z^2\right)\frac{s^2+6s+u^2}{s+u}+\mathcal{O}(1)\right] . \tag{7}$$

Question 7: Where does the appropriate 4-point vertex come from in the Standard Model Lagrangian?

Question 8* (Bonus Marks): Show the result (7).

The total matrix element behaves as

$$\mathcal{M}_{\mathrm{Tot}} = \frac{t}{v^2}+\mathcal{O}(1) , \tag{8}$$

where v is the Higgs vacuum expectation value.

Question 9: Show the result (8).

Question 10: Explain, in as much detail as possible, what is problematic about (8). At what energy scale will this problem occur?

Question 11: Show, in as much detail in possible, that introducing the Higgs boson to the Standard Model resolves the problem. What bound on the Higgs mass does this resolution require?

Question 12* (Bonus Marks): This calculation was performed in the unitary gauge. It is possible to obtain the same result in Lorentz gauge. Explain the difference between the gauge choices. Perform the calculation in the Lorentz gauge and show how the result it obtained there.

Question 13 (For Interest - No Marks): Do you think this analysis was strong enough evidence to invest in building the Large Hadron Collider? Is there a similar motivation for building a yet bigger collider? Should we do it?