Homework 1 - Introduction to Quantum Physics

Note:

It is often useful to remember the value $hc = 1.24 \times 10^{-6}\text{eV} \cdot \text{m}$.

Photoelectric Effect

Question 1 :

The wavelength of the photoelectric threshold for silver is 325nm. Determine the velocity of electron ejected from a silver surface by ultraviolet light of wavelength 253.6nm.

Solution:

According to Einsteins's equation, kinetic energy of the photoelectron

$$T = h\nu - \Phi = h\left(\nu - \nu_0\right),$$

where ν is the frequency of the incident photon and ν_0 is the threshold frequency. The corresponding energy for $\lambda = 253.6\text{nm}$ photon is

$$E = h\nu = \frac{hc}{\lambda} = \frac{1.241 \times 10^{-6}}{253.6 \times 10^{-9}}\text{eV} = 4,894\text{eV}.$$

In the same way, the threshold energy is

$$E_0 = 3.818\text{eV} \quad \Rightarrow \quad T = 1.076\text{eV}.$$

This energy translates into velocity as

$$T = \frac{1}{2}mv^2 = mc^2\frac{1}{2}\left(\frac{v}{c}\right)^2 = 0.511\text{eV} \times \frac{1}{2}\left(\frac{v}{c}\right)^2 = 1.076\text{eV} \quad \Rightarrow \quad v = 2.05 \times 10^{-3}c = 6.16 \times 10^5\frac{\text{m}}{\text{s}}.$$

Question 2:

A metal surface is illuminated with light of different wavelengths and the corresponding stopping potentials of the photoelectrons V , are summarized in a table below. Determine Planck's constant, the threshold wavelength and the work function.

Note: you may use computer to get the slope from a linear fit to the data.

λ[nm]	366	405	436	492	546	579
V[V]	1.48	1.12	0.93	0.92	0.36	0.24

Solution:

Recalling the equation for energy conservation

$$eV = \frac{hc}{\lambda} - W,$$

a plot of V against $1/\lambda$ must be a straight line. The slope of the line gives hc/e, hence h can be determined. The intercept multiplied by hc give W, the work function. The threshold frequency is given by $\nu_0 = W/h$. The results yield

$$\text{slope} = 1.24 \times 10^{-6} = \frac{hc}{e} \quad \Rightarrow \quad h = 6.6 \times 10^{-34} \frac{\mathrm{J}}{\mathrm{s}},$$
$$\text{intercept} = 1.5 \times 10^{-6} \mathrm{m}^{-1} = \frac{W}{hc} \quad \Rightarrow \quad W = 1.9\mathrm{eV},$$
$$\nu_0 = \frac{W}{h} = 4.545 \times 10^{14} \mathrm{s}^{-1} \quad \Rightarrow \quad \lambda_0 = \frac{c}{\nu_0} = 660\mathrm{nm}.$$

X-rays

Question 3:

If the minimum wavelength from an 80kV X-ray tube is 0.15Å, deduce a value for Planck's constant.

Solution:

Recall expression for the minimum wavelength we find

$$\lambda_{\min} = \frac{hc}{eV} \quad \Rightarrow \quad h = \frac{\lambda_{\min}}{c} eV = 6.4 \times 10^{-34} \mathrm{J} \cdot \mathrm{s}^{-1}.$$

Compton Scattering

Question 4:

In the Compton scattering, the photon of energy $E_0 = h\nu_0$ and momentum $p_0 = h\nu_0/c$ is scattered from a free electron of rest mass m. Show that

1. The scattered photon will have energy $E = E_0/\left[1 + \alpha\left(1 - \cos\phi\right)\right]$, where ϕ is the angle through which the photon is scattered and $\alpha = h\nu_0/mc^2$.
2. The kinetic energy acquired by the electron is $T = \alpha E_0 \left(1 - \cos\phi\right) / \left[1 + \alpha\left(1 - \cos\phi\right)\right]$
3. $\cot\left(\phi/2\right) = \left(1 + \alpha\right)\tan\theta$, where θ is the recoil angle of the electron.

Solution:

1. Let $E = h\nu$ be the energy of the scattered photon and $h\nu/c$ be its momentum. Energy conservation gives

$$h\nu_0 = h\nu + T,$$

where T is the electrons kinetic energy. Balancing momentum along and perpendicular to the direction of incidence yield

$$\frac{h\nu_0}{c} = \frac{h\nu}{c}\cos\phi + p_e\cos\theta \quad \text{and} \quad 0 = \frac{h\nu}{c}\sin\phi - p_e\sin\theta,$$

where p_e is the momentum of the scattered electron. Rearranging the momentum equations and adding the squares (just like we did in class) and using the relativistic equation for the electron's energy, we get

$$c^2 p_e^2 = T^2 + 2Tmc^2 = h^2\left(\nu_0^2 + \nu^2 - 2\nu_0\nu\cos\phi\right).$$

Finally, using the energy equation we find

$$E = \frac{E_0}{1 + \alpha\left(1 - \cos\phi\right)}.$$

2. From energy conservation

$$T = E_0 - E = E_0 - \frac{E_0}{1 + \alpha\left(1 - \cos\phi\right)} = \frac{\alpha E_0\left(1 - \cos\phi\right)}{1 + \alpha\left(1 - \cos\phi\right)}.$$

3. Going back to the momentum equation we can get

$$\cot\theta = \frac{\nu_0 - \nu\cos\phi}{\nu\sin\phi},$$

$$\tan\theta = \frac{\nu\sin\phi}{\nu_0 - \nu\cos\phi} = \frac{\sin\phi}{E_0/E - \cos\phi} = \frac{\sin\phi}{(\alpha+1)(1-\cos\phi)}$$

$$\Rightarrow \quad (\alpha+1)\tan\theta = \frac{\sin\phi}{1-\cos\theta} = \frac{2\sin(\phi/2)\cos(\phi/2)}{2\sin^2(\phi/2)} = \cot(\phi/2)$$

and using the result from (1) we find

$$\cot(\phi/2) = (1+\alpha)\tan\theta.$$

Question 5:

Calculate the maximum fractional frequency shift $\Delta\nu/\nu_0$ for an incident photon of wavelength $\lambda_0 = 1\text{Å}$ scattering off a proton initially at rest (Compton scattering analogue with proton instead of electron)

Solution:

The fractional frequency shift is

$$\frac{\Delta\nu}{\nu_0} = 1 - \frac{\lambda_0}{\lambda}.$$

recalling that

$$\lambda - \lambda_0 = \frac{h}{m_0 c}(1-\cos\phi) \quad \Rightarrow \quad \frac{\lambda}{\lambda_0} = 1 + \frac{h}{m_0 c\lambda_0}(1-\cos\phi),$$

we find

$$\frac{\Delta\nu}{\nu_0} = 1 - \frac{1}{1+\frac{h}{m_0 c\lambda_0}(1-\cos\phi)} = \left[\frac{m_0 c\lambda_0}{h(1-\cos\phi)} + 1\right]^{-1}.$$

Thus, the maximum fractional shift (i.e. when $\phi = \pi$) is

$$\left(\frac{\Delta\nu}{\nu_0}\right)_{\max} = \left[\frac{m_0 c\lambda_0}{2h} + 1\right]^{-1} = 2.645\times 10^{-5}.$$