Physics 1C - Tutorial 1

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1 Administrative Information

1. Eden Mautner, email: edenmau@post.bgu.ac.il
2. If you wish to attend the office hours, you should contact me by mail in advance so we can find a convenient time.
3. In this course, there will be a weekly tutorial and a weekly homework assignment. **There is no requirement to hand in your homework, but it is highly recommended to solve the assignment each week**.
4. The grade of this course is composed 100% from the exam's grade.

2 Introduction - Mathematical Tools

2.1 Derivatives

The derivative, calculated in a specific value, gives us the value of the tangent line for the same value. This indicates how rapid the change of the function is, and so we can think about the derivative as the "change rate" of the function.

Some useful derivative rules are:

$$\frac{d}{dx}\Big(f(x)\cdot g(x)\Big) = f'(x)\cdot g(x) + f(x)\cdot g'(x) \tag{1}$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{g(x)^2} \tag{2}$$

$$\frac{d}{dx}\Big(f(g(x))\Big) = f'(g)\cdot g'(x) \tag{3}$$

2.2 Integrals

Integration is the inverse operation of the derivative and can be thought as a sum of a continues variable. We can use integration to compute the area under a given graph (We will see this later in this tutorial).

2.3 Integrals and Derivatives of Useful Functions

2.3.1 Polynomials

$$f(x) \equiv a_n \cdot x^n + a_{n-1} \cdot x^{n-1} + ... + a_2 \cdot x^2 + a_1 \cdot x + a_0 \tag{4}$$

$$f'(x) = a_n \cdot nx^{n-1} + a_{n-1} \cdot (n-1)x^{n-2} + ... + a_2 \cdot 2x + a_1 \tag{5}$$

$$\int f(x)dx = a_n \cdot \frac{x^{n+1}}{n+1} + a_{n-1} \cdot \frac{x^n}{n} + ... + a_2 \cdot \frac{x^3}{3} + a_1 \cdot \frac{x^2}{2} + a_0 \cdot x + C \tag{6}$$

2.3.2 Trigonometric Functions

$$f(x) = \sin(ax) \;\rightarrow\; f'(x) = a \cdot \cos(ax) \;\;,\;\; \int f(x)dx = -\frac{1}{a} \cdot \cos(ax) + C \tag{7}$$

$$f(x) = \cos(ax) \;\rightarrow\; f'(x) = -a \cdot \sin(ax) \;\;,\;\; \int f(x)dx = \frac{1}{a} \cdot \sin(ax) + C \tag{8}$$

2.3.3 Exponential Functions

$$f(x) = e^{ax} \;\rightarrow\; f'(x) = a \cdot e^{ax} \;\;,\;\; \int f(x)dx = \frac{1}{a} \cdot e^{ax} + C \tag{9}$$

3 Physical Definitions

3.1 Position, Path and Displacement

Position: The location of the object in respect to the origin.

Path: The total distance the object has passed.

Displacement: The total change in the position of the object. For example, If the object started at location x_i and got to location x_f, the displacement is given by:

$$\Delta x = x_f - x_i \tag{10}$$

3.2 Velocity

Average velocity is given by:

$$\bar{v} = \frac{\Delta x}{\Delta t} \tag{11}$$

Instantaneous velocity is given by:

$$v(t) = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx(t)}{dt} \tag{12}$$

3.3 Acceleration

Acceleration is the change in velocity, therefore, it is given by:

$$a(t) = \frac{dv(t)}{dt} = \frac{d^2x(t)}{dt^2} \tag{13}$$

3.4 Equations Of Motion

Motion with **constant velocity** means that there is no acceleration, therefore $a = 0$. In this case the position equation is given by:

$$x(t) = x_0 + v \cdot t \tag{14}$$

Motion with **constant acceleration** is given by:

$$x(t) = x_0 + v_0 \cdot t + \frac{1}{2}a \cdot t^2 \tag{15}$$

In general, when the velocity is given, and it is not constant (depends on time), we can find the position equation by integration:

$$x(t) = x_0 + \int_{t_i}^{t_f} v(t)dt \tag{16}$$

4 Question 1

חלקיק נע לאורך ציר x על פי הביטוי: $x = 6 + 2t + 5t^2$

א. מצא/י את תאוצתו של הגוף?

ב. מהי מהירותו ההתחלתית (ב- t=0) של הגוף, ומהו מיקומו ההתחלתי?

ג. מצא/י ביטוי כללי למהירותו של הגוף כתלות בזמן t.

ד. מהי מהירותו בזמן t = 4 sec ?

Solution

1. + 3. The acceleration of the particle can be found by calculating the second derivative of the position equation.
By calculating the first derivative, we find the velocity:

$$v(t) = \dot{x}(t) = 2 + 10t \ [\frac{m}{s}] \tag{17}$$

now by calculating the derivative of the velocity we find the acceleration:

$$a(t) = \dot{v}(t) = \ddot{x}(t) = 10 \ [\frac{m}{s^2}] \tag{18}$$

2. To find the initial location and the initial velocity of the particle, all we need to do is plug $t = 0$ into $x(t)$ and $v(t)$, and so we get:

$$x(t=0) = 6 + 2 \cdot 0 + 5 \cdot 0^2 = 6 \ [m] \quad , \quad v(t=0) = 2 + 10 \cdot 0 = 2 \ [\frac{m}{s}] \tag{19}$$

4. To find the velocity of the particle at $t = 4$, we need to plug in this value into $v(t)$, and so we get:

$$v(t=4) = 2 + 10 \cdot 4 = 42 \ [\frac{m}{s}] \tag{20}$$

5 Question 2

$x = 30m + 16\frac{m}{sec}t + 3\frac{m}{sec^2}t^2$:תנועה של מטוס מתוארת ע"י המשוואה

א) מהי המהירות הממוצעת לאחר 2 דקות ?

ב) מהי המהירות הממוצעת בין הדקה הראשונה לשניה ?

ג) מה המהירות הרגעית בזמן t = 30sec ?

Solution

1. As we have discussed in the introduction, the average velocity is defined as $\bar{v} = \frac{\Delta x}{\Delta t}$. Using this equation, we find that the average velocity after 2 minutes is:

$$\bar{v} = \frac{x(120) - x(0)}{120 - 0} = \frac{45150 - 30}{120} = 376\ [\frac{m}{s}] \tag{21}$$

2. Using the exact same logic, we find:

$$\bar{v} = \frac{x(120) - x(60)}{120 - 60} = \frac{45150 - 11790}{60} = 556\ [\frac{m}{s}] \tag{22}$$

3. We recall that the instantaneous velocity is given by calculating the derivative of the position equation (with respect to time).

$$v(t) = \dot{x}(t) = 16 + 6t \quad \rightarrow \quad v(30) = 16 + 6 \cdot 30 = 196\ [\frac{m}{s}] \tag{23}$$

6 Question 3

$Z(t) = 16te^{-t}$:חלקיק נע לאורך ציר z פונקצית המיקום שלו נתונה ע"י

מיקום החלקיק נתון ביחידות של מטרים

מהו מרחקו של החלקיקי מהראשית כאשר הוא נעצר? באיזה זמן זה קורה?

כדי לקבל תחושה לגבי התנועה, שרטטו אותה כפונקציה של הזמן.

Solution

The particle will stop when its velocity reaches zero, therefore:

$$v(t) = \dot{z}(t) = 16e^{-t} - 16te^{-t} = 16e^{-t}(1 - t) \tag{24}$$

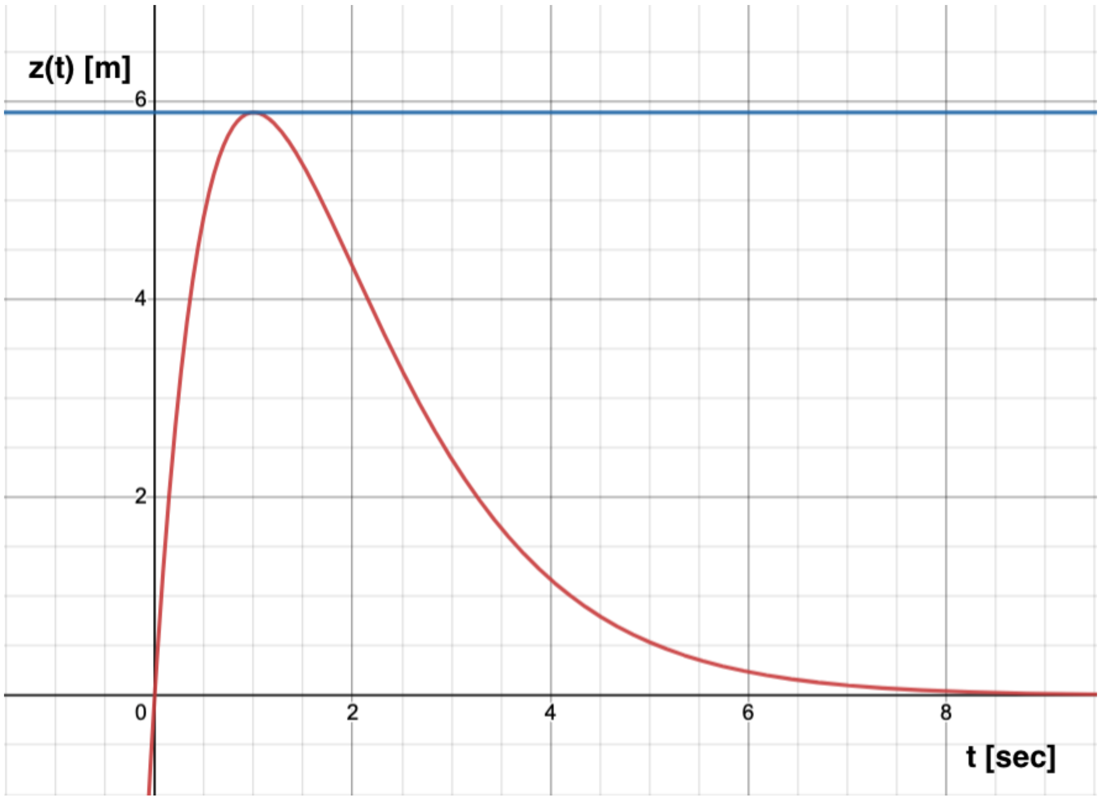
Solving for $v(t) = 0$ we find:

$$16e^{-t}(1 - t) = 0 \quad \rightarrow \quad t = 1 \tag{25}$$

To find the distance of the particle from the origin, we need to plug $t = 1$ into $z(t)$:

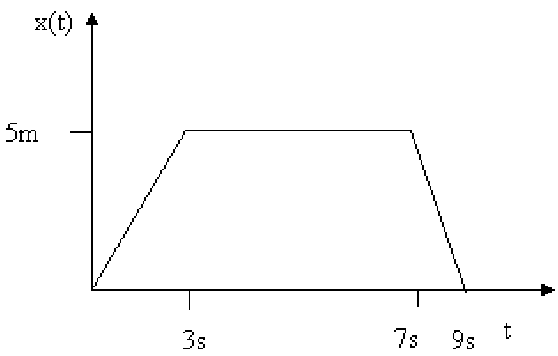
$$z(1) = 16 \cdot 1 \cdot e^{-1} = 16e^{-1} \ [m] \tag{26}$$

The graph of $z(t)$ is presented below:



7 Question 4

נתון גרף המתאר תנועה של גוף על ציר ה x מצא את x כפונקציה של הזמן לכל איזור.
שרטט גרף של מהירות כפונקציה של הזמן



Solution

We can see that the solution can be divided into 3 sections.

- $0 < t < 3$ [s]
- $3 < t < 7$ [s]
- $7 < t < 9$ [s]

For each section, we need to find the corresponding equation for $x(t)$. In general we know that the equation is in the form of:

$$x(t) = A \cdot t + x_0 \quad , \quad A = \frac{\Delta x}{\Delta t} = v \quad \rightarrow \quad x(t) = v \cdot t + x_0 \tag{27}$$

Now we can calculate the position equation for each section.

Section 1 : We immediately see that $x_0 = 0$. All that is left to calculate is the slope v:

$$A = v = \frac{\Delta x}{\Delta t} = \frac{5-0}{3-0} = \frac{5}{3} \; [\frac{m}{s}] \tag{28}$$

and so we find that:

$$x(t) = \frac{5}{3} \cdot t \quad , \quad 0 < t < 3 \; [s] \tag{29}$$

Section 2 : We see from the graph that in this time interval there is no change in position, therefore:

$$x(t) = 5 \; [m] \quad , \quad 3 < t < 7 \; [s] \tag{30}$$

Section 3 : We can start by calculating the slope v for this section:

$$A = v = \frac{\Delta x}{\Delta t} = \frac{0-5}{9-7} = -\frac{5}{2} \; [\frac{m}{s}] \tag{31}$$

and by calculating we can that $x_0 = \frac{45}{2}$ $[m]$, therefore, we finally obtain:

$$x(t) = -\frac{5}{2} \cdot t + \frac{45}{2} \quad , \quad 7 < t < 9 \; [s] \tag{32}$$

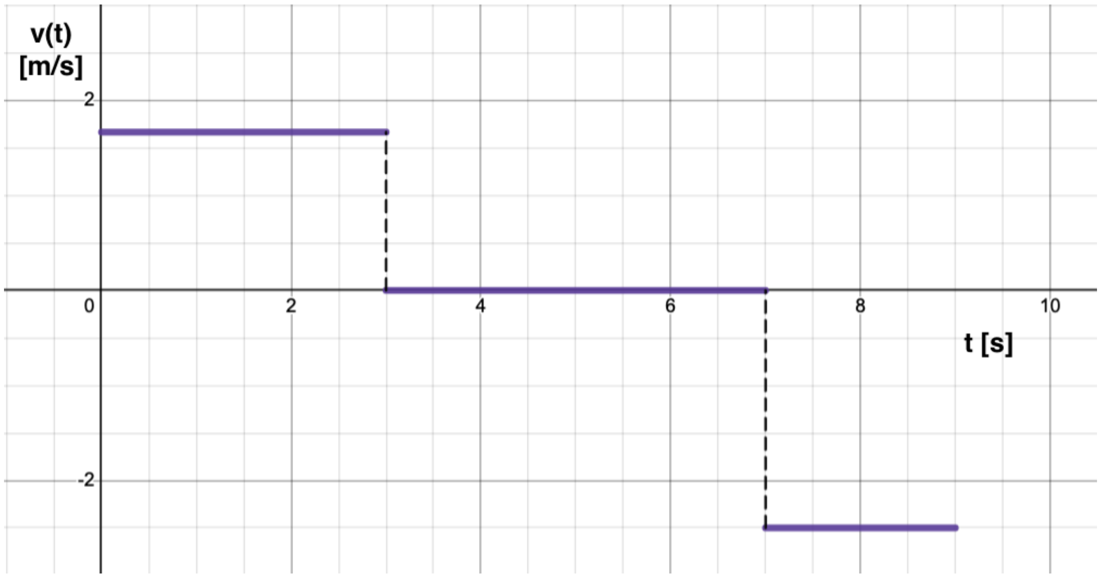
So we can conclude and write:

$$x(t) = \begin{cases} \frac{5}{3} \cdot t & , \quad 0 < t < 3 \; [s] \\ 5 & , \quad 3 < t < 7 \; [s] \\ -\frac{5}{2} \cdot t + \frac{45}{2} & , \quad 7 < t < 9 \; [s] \end{cases} \tag{33}$$

The velocity as a function of time can be found by calculating the derivatives of $x(t)$:

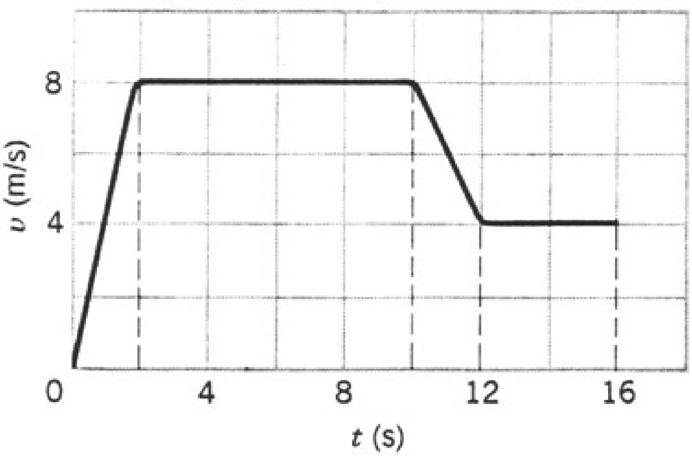
$$v(t) = \dot{x}(t) = \begin{cases} \frac{5}{3} & , \quad 0 < t < 3 \; [s] \\ 0 & , \quad 3 < t < 7 \; [s] \\ -\frac{5}{2} & , \quad 7 < t < 9 \; [s] \end{cases} \tag{34}$$

so we can draw the following graph of $v(t)$:



8 Question 5

מהירותו של רץ באימון נתונה בגרף הבא:



א. מהו המרחק אותו רץ הרץ באימונו ?
ב. בטא/י את $x(t)$ (מיקום הרץ כתלות בזמן), שרטט/י גרף של $x(t)$ כנגד t.
ג. מהי המהירות הממוצעת של הרץ ב-10 השניות הראשונות ?
ד. שרטט באופן סכמטי גרף המתאר את תאוצת הרץ כפונק' של הזמן?

Solution

1. We know that the total displacement is given by $x = \int v dt$, which is the area under the curve. In this case, it is straightforward to calculate the areas under the graph.

$$Area = Triangle + Rectangle\ 1 + Trapez + Rectangle\ 2 \tag{35}$$

so by calculating, we get:

$$x = \frac{2 \cdot 8}{2} + 8 \cdot 8\ + \frac{(8+4) \cdot 2}{2} + 4 \cdot 4 = 100\ [m] \tag{36}$$

2. First, we would like to find the velocity equation, $v(t)$, for each section. The general equation is given by $v(t) = a \cdot t + v_0$. The corresponding accelerations for each section are given by:

$$a = \frac{\Delta v}{\Delta t} = \begin{cases} 4\ [\frac{m}{s^2}] & , \quad v_0 = 0\ [\frac{m}{s}] \quad , \quad 0 < t < 2\ [s] \\ 0\ [\frac{m}{s^2}] & , \quad v_0 = 8\ [\frac{m}{s}] \quad , \quad 2 < t < 10\ [s] \\ -2\ [\frac{m}{s^2}] & , \quad v_0 = 28\ [\frac{m}{s}] \quad , \quad 10 < t < 12\ [s] \\ 0\ [\frac{m}{s^2}] & , \quad v_0 = 4\ [\frac{m}{s}] \quad , \quad 12 < t < 16\ [s] \end{cases} \tag{37}$$

so in total, the velocity function is given by:

$$v(t) = \begin{cases} 4 \cdot t & , \quad 0 < t < 2\ [s] \\ 8\ [\frac{m}{s}] & , \quad 2 < t < 10\ [s] \\ -2 \cdot t + 28 & , \quad 10 < t < 12\ [s] \\ 4\ [\frac{m}{s}] & , \quad 12 < t < 16\ [s] \end{cases} \tag{38}$$

Now to find the position equation, we need to integrate each section. We notice that the runner started from location $x = 0$. We get:

$$x(t) = \int v(t)dt = \begin{cases} 2t^2 & , \quad 0 < t < 2\ [s] \\ 8t + c_1 & , \quad 2 < t < 10\ [s] \\ -t^2 + 28t + c_2 & , \quad 10 < t < 12\ [s] \\ 4t + c_3 & , \quad 12 < t < 16\ [s] \end{cases} \tag{39}$$

Now, we can find c_1, c_2, and c_3 using the fact that we know the displacement at different times:

$$x(2) = 8\ [m] \quad \rightarrow \quad c_1 = -8\ [m] \tag{40}$$

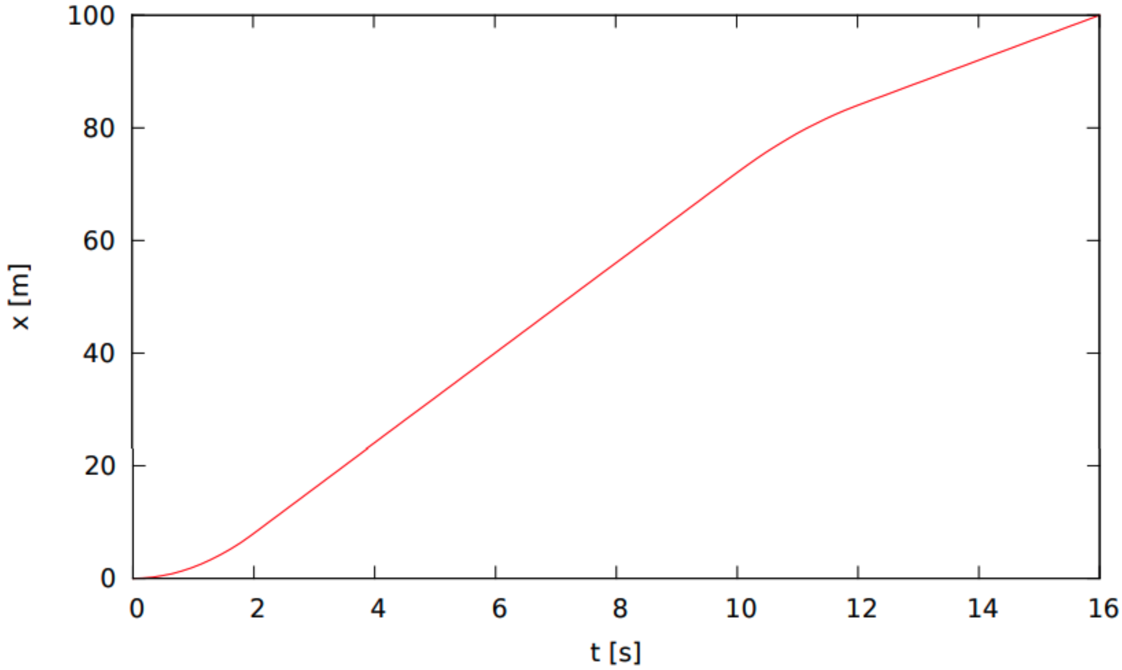
$$x(10) = 72\ [m] \quad \rightarrow \quad c_2 = -108\ [m] \tag{41}$$

$$x(12) = 84\ [m] \quad \rightarrow \quad c_3 = 36\ [m] \tag{42}$$

And so, in total:

$$x(t) = \begin{cases} 2t^2 & , \quad 0 < t < 2\ [s] \\ 8t - 8 & , \quad 2 < t < 10\ [s] \\ -t^2 + 28t - 108 & , \quad 10 < t < 12\ [s] \\ 4t + 36 & , \quad 12 < t < 16\ [s] \end{cases} \tag{43}$$

A schematic graph of $x(t)$ is given by:



3. The average velocity is given by $\bar{v} = \frac{\Delta x}{\Delta t}$, so in this case we get:

$$\bar{v} = \frac{72 - 0}{10 - 0} = 7.2 \; [\frac{m}{s}] \tag{44}$$

4. Using what we calculated in section one of this question, the acceleration graph is given by: