Physics 1C - Tutorial 2

Eden Mautner

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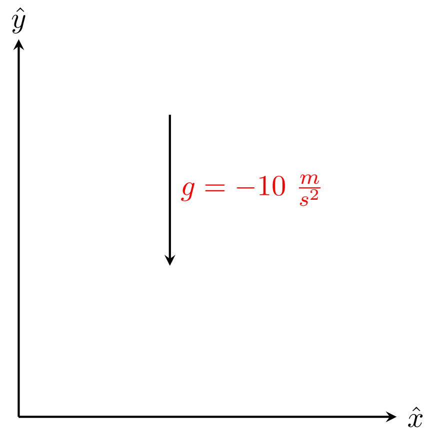
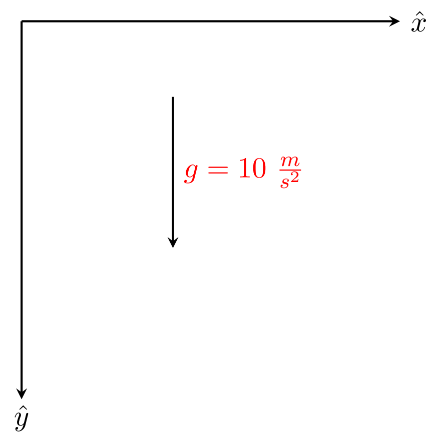
1 Reminder

Free fall describes the motion of an object towards the center of the planet with constant acceleration. Usually, we would like to define the positive y-axis in the upwards direction so that the acceleration is $g = -9.8\ \frac{m}{s^2} \approx -10\ \frac{m}{s^2}$. We should notice that if we define the positive y-axis inwards, for example, towards the center of the earth, then the acceleration will be positive. The difference can be seen in the following illustrations:





In general, an object in free fall has an acceleration of g. Therefore the velocity of the object can be calculated by integration as follows:

$$a = g \quad \rightarrow \quad v(t) = \int a dt = \int g dt = gt + v_0 \tag{1}$$

where v_0 is the initial velocity and can be found from initial conditions.
By integrating once more, we get the position equation:

$$y(t) = \int v(t) dt = \int (gt + v_0) dt = \frac{1}{2} g t^2 + v_0 t + y_0 \tag{2}$$

similarly to v_0, here y_0 is the initial position of the object which can be found by using the initial conditions.

2 Question 1

גוף נזרק כלפי מעלה במהירות של
$v_0 = 30\ m/s$ מטר לשניה.
א. היכן ימצא הגוף לאחר $t_1 = 2\ s$ שניות.
ב. מה תהיה מהירותו לאחר $t_2 = 2.5\ s$ שניות.
ג. כמה זמן תימשך עלייתו.
ד. מהו הגובה המקסימלי במסלולו.
ה. באיזה מהירות יגיע לנקודת הזריקה.
ו. לאחר כמה זמן הגוף יהיה $d = 10\ m$ מתחת לנקודת הזריקה.

Solution

1. Firstly, we want to define the axis on which we are working. Let's define the positive y axis so it's pointing upwards. Now we can continue by writing the equations of motion, that is, $v(t)$ and $y(t)$:

$$v(t) = v_0 + gt \quad , v_0 = 30\ \frac{m}{s} \quad , \quad g = -10\ \frac{m}{s^2} \quad \rightarrow \quad v(t) = 30 - 10t \tag{3}$$

now, to find $y(t)$ we can integrate $v(t)$:

$$y(t) = y_0 + 30t - 5t^2 \tag{4}$$

If we wish to find the location of the object at $t_1 = 2\ s$, all that is left to do is plug it into $y(t)$:

$$y(2) = y_0 + 30 \cdot 2 - 5 \cdot 2^2 = y_0 + 40\ m \tag{5}$$

so we see that after two seconds, the object will be 40 meters above the starting point.

2. To find the velocity after two and a half seconds, all we need to do is plug $t = \frac{5}{2}s$ into $v(t)$:

$$v(\frac{5}{2}) = 30 - 10 \cdot \frac{5}{2} = 5\ \frac{m}{s} \tag{6}$$

3. To find the travel time of the object, all we need to do is find when the velocity is zero:

$$v(t) \equiv 0 \quad \rightarrow \quad v(t) = 30 - 10t = 0 \quad \rightarrow \quad t = 3s \tag{7}$$

4. From the previous subsection, we know that the object will reach the maximal height after $t = 3s$. So we find:

$$h_{max} = y(3) = y_0 + 30 \cdot 3 - 5 \cdot 3^2 = y_0 + 45\ m \tag{8}$$

5. To answer this, we need to find when the object returned to y_0.

$$y(t) \equiv y_0 \quad \rightarrow \quad y(t) = y_0 + v_0 t + \frac{1}{2} g t^2 \equiv y_0 \tag{9}$$

so we get:

$$t(v_0 + \frac{1}{2}gt) = 0 \quad \rightarrow \quad t_1 = 0 \;\; , \;\; t_2 = \frac{-2v_0}{g} \tag{10}$$

We can see that $t_1 = 0$ is the time at which the object started its journey. Now, by plugging in $v_0 = 10\ m$, $g = -10\ \frac{m}{s^2}$ we find that:

$$t_2 = \frac{-2 \cdot 30}{-10} \quad \rightarrow \quad t_2 = 6\ s \tag{11}$$

Now, all that is left to do is plug t_2 into the velocity equation, $v(t)$, and find the velocity of the object when it returns to the starting point.

$$v(6) = 30 - 10 \cdot 6 = -30\ \frac{m}{s} \tag{12}$$

6. Similarly to the previous subsection, to answer this question, we want to know when the object arrived to $y = y_0 - 10\ m$:

$$y(t) \equiv y_0 - 10 \quad \rightarrow \quad y(t) = y_0 + v_0 t + \frac{1}{2}gt^2 \equiv y_0 - 10 \tag{13}$$

Plugging in v_0 and g we get:

$$y(t) = y_0 + 30t - 5t^2 \equiv y_0 - 10 \quad \rightarrow \quad t^2 - 6t - 2 = 0 \tag{14}$$

so the solutions are:

$$t_{1,2} = \frac{6 \pm \sqrt{36 - 4 \cdot 1 \cdot (-2)}}{2} = 3 \pm \sqrt{11}\ s \tag{15}$$

finally, we can omit the negative solution since we know that $t \geq 0$. Therefore, we conclude that the correct solution is:

$$t = 3 + \sqrt{11}\ s \tag{16}$$

3 Question 2

סטודנט זורק צרור מפתחות כלפי מעלה , לעבר השותפה שלו, הנשענת על אדן החלון הדירה , בגובה $4.00m$ מעליו.
השותפה תפסה את המפתח לאחר פרק זמן של $1.50s$.
א. באיזו מהירות התחלתית נזרק המפתח?
ב. ברגע שהשותפה תפסה את המפתח, מה היתה מהירותו?

Solution

1. Let's assume that at $t = 0$ the keys are located at the origin, that is, $y_0 = 0$. We know that a general equation for the location of the keys at time t is given by:

$$y(t) = y_0 + v_0 t + \frac{1}{2}gt^2 \tag{17}$$

plugging in $y_0 = 0$ and $g = -10\frac{m}{s^2}$ we get the following equation:

$$y(t) = v_0 t - 5t^2 \tag{18}$$

now, since we know that the student's roommate is located 4 m above and that she took hold of the keys 1.5 s after the student tossed them in the air, we know that:

$$y(1.5) \equiv 4 \quad \rightarrow \quad \frac{3}{2} \cdot v_0 - 5 \cdot (\frac{3}{2})^2 = 4 \quad \rightarrow \quad v_0 = \frac{61}{6} \; \frac{m}{s} \tag{19}$$

2. To answer this part of the question, we need to find the velocity equation. By calculating the derivative of $y(t)$ we know that:

$$v(t) = \dot{y}(t) = v_0 - 10t \tag{20}$$

by plugging $t = 1.5\ s$ and $v_0 = \frac{61}{6}\ \frac{m}{s}$ we find:

$$v(1.5) = \frac{61}{6} - 10 \cdot \frac{3}{2} \quad \rightarrow \quad v(1.5) = -\frac{29}{6} \; \frac{m}{s} \tag{21}$$

We can learn from the minus sign of the velocity that she grabbed the keys on their way down.

4 Question 3

אגוז קוקוס מופל מגובה $73m$
$1.2s$ אחר כך אגוז קוקוס שני מושלך למטה עם מהירות התחלתית כלשהי. שני האגוזים מגיעים לקרקע באותו זמן
מהי מהירותו ההתחלתית של האגוז השני?

Solution

In this question, our goal is to find an expression for the time it took each coconut to reach the ground. Let's start by working on the kinematics of the first coconut. We will define the y axis in the upwards direction and the ground as the origin, meaning $y = 0$.

Coconut 1: We know that the general location equation is given by:

$$y(t) = y_0 + v_0 t + \frac{1}{2} g t^2 \tag{22}$$

and since we are interested to find t for which $y(t) = 0$ we want to solve the following equation:

$$y(t) \equiv 0 \quad \rightarrow \quad y_0 + v_0 t + \frac{1}{2} g t^2 = 0 \tag{23}$$

Now, by plugging $v_0 = 0$ into $y(t)$ we get the following equation:

$$y_0 + \frac{1}{2} g t^2 = 0 \quad \rightarrow \quad t = \pm\sqrt{-\frac{2 \cdot y_0}{g}} \tag{24}$$

After omitting the negative answer and plugging $g = -10\frac{m}{s^2}$, $y_0 = 73$, we find that the time it took the first coconut to hit the ground is:

$$t = \sqrt{\frac{146}{10}} \approx 3.82\ s \tag{25}$$

Coconut 2: Once again, we know that the general location equation for the second coconut is given by:

$$y(t) = y_0 + v_0 t + \frac{1}{2}gt^2 \tag{26}$$

And since we know that both coconuts reached the ground simultaneously, and we know that coconut two was thrown 1.2 seconds after coconut 1, we know that $t_2 = 3.82 - 1.2 = 2.62\ s$ was the total time that coconut 2 was in the air. This means that the following equation must be true:

$$y(t_2) \equiv 0 \quad \rightarrow \quad y_0 + v_0 t_2 + \frac{1}{2}gt_2^2 = 0 \tag{27}$$

solving for v_0 we get:

$$v_0 = -\Big(\frac{y_0}{t_2} + \frac{1}{2}gt_2\Big) \tag{28}$$

Now, if we plug $y_0 = 73m$, $g = -10\ \frac{m}{s^2}$ and $t_2 = 2.62\ s$ into the equation above, we will find the initial velocity of coconut 2:

$$v_0 = -\Big(\frac{73}{2.62} - 5 \cdot 2.62\Big) \quad \rightarrow \quad v_0 = -14.76\ \frac{m}{s} \tag{29}$$

We notice that the velocity is negative. This is exactly what we expected since positive velocity would mean that the coconuts couldn't have reached the ground simultaneously.

5 Question 4

מכונית עומדת ברמזור, מרגע שהרמזור מתחלף לירוק עוקף אותה רוכב אופניים. לאופניים מהירות קבועה של 30 קמ"ש. המכונית מאיצה בתאוצה קבועה של 5 מטר לשניה בריבוע.
א. לאחר כמה זמן יפגשו המכונית והאופניים?
ב. מה יהיה המרחק של נקודת העקיפה מנקודת הזינוק של המכונית?

Solution

1. The position equation of the bicycle rider is quite simple since we know that $x_0 = 0$ and that he has a constant velocity of $v_b = 30\frac{km}{h}$. Therefore:

$$x_b(t) = v_b t \tag{30}$$

On the other hand, we know that the car also starts from $x_0 = 0$, but with no initial velocity ($v_0 = 0$) and with a constant acceleration of $a = 5\frac{m}{s^2}$. Therefore, the position equation of the car is given by:

$$x_c(t) = \frac{1}{2}at^2 \tag{31}$$

Now, to find how long it took the car to get past the bicycle rider, all we need to do is solve the following equation:

$$x_b(t) = x_c(t) \quad \rightarrow \quad v_b t = \frac{1}{2}at^2 \quad \rightarrow \quad t = \frac{2v_b}{a} \tag{32}$$

Now we must notice that v_b is in units of $\frac{km}{h}$ so we would like to convert these to units of $\frac{m}{s}$:

$$v_b = 30\ \frac{km}{h} = 30 \cdot \frac{1000m}{3600s} \quad \rightarrow \quad v_b = \frac{25}{3}\frac{m}{s} \tag{33}$$

So plugging this back into t, we get:

$$t = \frac{2}{5} \cdot \frac{25}{3} = \frac{10}{3} s \tag{34}$$

2. To find the answer to this question, we need to plug in the relevant values to either of the position equations we found previously. For instance, let's plug the relevant values into $x_b(t)$. Doing so, we find that the distance at which the car passed the bicycle rider is:

$$x_b(t = \frac{10}{3} s) = \frac{25}{3} \cdot \frac{10}{3} \quad \rightarrow \quad x_b = \frac{250}{9}\ m \tag{35}$$

6 Question 5

ניתן לבנות מערכת פשוטה למדידת זמן תגובה בעזרת קרטון ועט. מסמנים על הקרטון שתי נקודות, אחת בחלק העליון ואחת בחלק התחתון. חברך/תך מחזיק/ה את הקרטון בנקודה העליונה, ואילו ידיך נמצאות במצב הכן, קרוב ככל הניתן לנקודה התחתונה. ברגע מסוים, החבר/ה עוזב/ת את הקרטון, ומטרתך לתפוס אותו בהקדם האפשרי. בעזרת נקודת התפיסה ניתן לברר את זמן תגובתך. היכן יש למקם את השנתות המציינות 50, 100, 150, 200, 250 מילישניות? (בהינתן $g = 10\frac{m}{s^2}$)

Solution

In this question, thinking about the positive y axis pointing downwards can be easier. Using the fact that the object is in free fall and since the acceleration is given, we can calculate the distance that the object has fallen in $t = [50, 100, 150, 200, 250]\ ms$. This distance is the same distance we should place the scale marks.

The free fall equation, with $y_0 = 0$ and $v_0 = 0$ is:

$$y(t) = \frac{1}{2} g t^2 \tag{36}$$

Plugging into the above equation the different values of t, we find:

- $t = \ 50ms = \ 50 \cdot 10^{-3} s \quad \rightarrow \quad y(t = 0.05s) = 0.0125\ m$
- $t = 100ms = 100 \cdot 10^{-3} s \quad \rightarrow \quad y(t = 0.1s) = 0.05\ m$
- $t = 150ms = 150 \cdot 10^{-3} s \quad \rightarrow \quad y(t = 0.15s) = 0.1125\ m$
- $t = 200ms = 200 \cdot 10^{-3} s \quad \rightarrow \quad y(t = 0.2s) = 0.2\ m$
- $t = 250ms = 250 \cdot 10^{-3} s \quad \rightarrow \quad y(t = 0.25s) = 0.3125\ m$