Physics 1C - Homework 1 Question 3

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אצן אולימפי מקווה לסיים את ריצת 5000 המטרים בפחות מ- 13 דקות. אם לאחר 11 דקות של ריצה במהירות קבועה נשארו לו 800 מטרים לסוף המסלול, ותאוצתו המקסימאלית היא 0.2 מטר לשניה בריבוע, מהו הזמן המינימאלי שעליו להאיץ על מנת לעמוד בזמן של 13 דקות?

Solution

Before we dive into the solution of this question, we should try and think about what we expect to see while solving the question and how we can simplify it. The first thing we can do is divide the problem into three parts:

1. First 11 minutes of the run. At this part, the runner runs with constant velocity, let's say, v_1.
2. At 2nd part of the run the runner runs with his maximal acceleration, $a_{max} = 0.2\ \frac{m}{s^2}$. We know that in the case of maximal acceleration the time needed to accelerate will be minimal. Let's define this minimal time as t_{min}.
3. At the last part of the run, after the runner stopped accelerating, he had a new constant velocity, let's say, v_3.

Now, one thing we can understand immediately is that the runner had $\tilde{t} = 120\ s$ to finish the last $800\ m$ of the run. This means that the total time it took the runner to finish part 3 of the run is $t_3 = \tilde{t} - t_{min}$

Now, in total we know that:

$$\Delta x_{total} = \Delta x_{1st\ part} + \Delta x_{2nd\ part} + \Delta x_{3rd\ part} \tag{1}$$

From reading the question we know that

$$\Delta x_{total} = 5000\ m \quad , \quad \Delta x_{1st\ part} = 4200\ m$$

This reduces Eq.(1) to the following equation:

$$\Delta x_{2nd\ part} + \Delta x_{3rd\ part} = 800\ m \tag{2}$$

Now we have finally reached a point that we can write the relevant equations for the second and third part of the run.

First, we know that the second part of the run started with an initial velocity of v_0 and with a constant acceleration a_{max}. In addition, we know that the run lasted for t_{min}. Therefore we can write:

$$\Delta x_{2nd\ part} = v_0 t_{min} + \frac{1}{2} a_{max} t_{min}^2 \tag{3}$$

To find v_0, all we need to do is understand that the initial velocity of the 2nd part is actually the constant velocity of the 1st part of the run. Therefore:

$$v_0 = \frac{\Delta x_{1st\ part}}{t_{1st\ part}} = \frac{4200\ m}{11\ min} = \frac{4200\ m}{11 \cdot 60\ s} \quad \rightarrow \quad v_0 \approx 6.36\ \frac{m}{s} \tag{4}$$

Now, before plugging v_0 straight into Eq.(3), we know that the velocity of the runner at the end of the 2nd part is exactly the constant velocity of the runner at the 3rd part of the run, v_3, and that it is given by:

$$v_3 = v_0 + a_{max} t_{min} \tag{5}$$

If so, we can write the equation for the last part of the run:

$$\Delta x_{3rd\ part} = v_3 t_3 = v_3(\tilde{t} - t_{min}) \tag{6}$$

Now we can plug all into Eq.(2):

$$\Delta x_{2nd\ part} + \Delta x_{3rd\ part} = v_0 t_{min} + \frac{1}{2} a_{max} t_{min}^2 + v_3(\tilde{t} - t_{min}) \tag{7}$$

and by plugging v_3 to the equation above we have:

$$\Delta x_{2nd\ part} + \Delta x_{3rd\ part} = v_0 t_{min} + \frac{1}{2} a_{max} t_{min}^2 + (v_0 + a_{max} t_{min})(\tilde{t} - t_{min}) \tag{8}$$

After arranging the equation we are left with:

$$800\ m = \Delta x_{2nd\ part} + \Delta x_{3rd\ part} = -\frac{1}{2} a_{max} t_{min}^2 + a_{max} \tilde{t} \cdot t_{min} + v_0 \tilde{t} \tag{9}$$

We can now plugin the following values, which we have found previously:

$$a_{max} = 0.2\ \frac{m}{s^2} \quad , \quad \tilde{t} = 2\ min = 120\ s \quad , \quad v_0 = 6.36\ \frac{m}{s}$$

And now, all that is left for us to do is solve this equation:

$$-\frac{1}{10} t_{min}^2 + 24 t_{min} + 763.2 = 800 \quad \rightarrow \quad t_{min}^2 - 240 t_{min} + 368 = 0 \tag{10}$$

The solutions for this equation are

$$t_{min} = 1.54\ s \quad , \quad t_{min} = 238.45\ s$$

Now, since we are interested in the minimal time and we expect the time to be under two minutes, we can omit the second solution, that is $t_{min} = 238.45\ s$.

Therefore, we conclude that the minimal time that the runner has to accelerate, with an acceleration of $a = 0.2\ \frac{m}{s^2}$, if he wishes to finish in 13 minutes is

$$t_{min} = 1.54\ s$$