Physics 1C - Tutorial 3

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1 Reminder

1.1 Vectors

Definition: In physics, a **vector** refers to an object which can be described by its size and direction. Examples of vectors from our daily life are:

- Displacement.
- Velocity.
- Angular Velocity.
- Acceleration.
- Force.

Vectors can be **added or subtracted** from each other. **Multiplying** a vector by a scalar can stretch or contract the original vector.

1.2 Cartesian and Polar Coordinate Systems

In general, when we approach a kinematics question in physics, we **must** define the coordinate system in which we are working. We had seen, in the last tutorial, when we discussed free fall (1D), that it was crucial to define the coordinate system in each problem we solved. There is no exception when solving problems in 2D or 3D. In 2D, we define the basis vectors of the coordinate system as

$$\hat{x} \equiv \hat{\imath} \equiv (1,0,0) \quad , \quad \hat{y} \equiv \hat{\jmath} \equiv (0,1,0), \quad \hat{z} \equiv \hat{k} \equiv (0,0,1)$$

Now, we can use two basis vectors to span the entire 2D space. This means that every point on the 2D cartesian coordinate system can be described in the following way:

$$\vec{r} = \begin{pmatrix} r_x \\ r_y \end{pmatrix} = \begin{pmatrix} r_x & r_y \end{pmatrix} = r_x\hat{x} + r_y\hat{y} = r_x\hat{\imath} + r_y\hat{\jmath} \tag{1}$$

We can also present the vector in a polar coordinate system. If we think intuitively about this, we understand that all that is required to describe a point in 2D space successfully is the distance from the point, that is r, and the angle between r and the positive x axis, known as θ. Therefore, in a polar coordinate system, $\vec{r}$ can be described by:

$$r = |\vec{r}\,| = \sqrt{r_x^2 + r_y^2} \quad , \quad \theta = \arctan\left(\frac{r_y}{r_x}\right) \quad \rightarrow \quad \vec{r} = (r\ ,\ \theta) \tag{2}$$

1.3 Motion in 2D

As we already know, the basis vectors $\hat{x}$ and $\hat{y}$ are perpendicular to each other. As a result, we can deal with each axis separately. Therefore, all prior definition hold for each axis on its own. For instance, if we have a position vector

$$\vec{r} = (r_x \ , \ r_y) = r_x\hat{x} + r_y\hat{y} \tag{3}$$

then the velocity can be found by calculating the derivative of each axis:

$$\vec{v} \equiv \dot{\vec{r}} = (\dot{r}_x \ , \ \dot{r}_y) = (v_x \ , \ v_y) = v_x\hat{x} + v_y\hat{y} \tag{4}$$

To find the acceleration, all we need to do is take the derivative of $\vec{v}$.

$$\vec{a} \equiv \dot{\vec{v}} = (\dot{v}_x \ , \ \dot{v}_y) = (a_x \ , \ a_y) = a_x\hat{x} + a_y\hat{y} \tag{5}$$

2 Question 1

א. נתון וקטור בהצגה קרטזית $\vec{r} = (3, 4)$ הצג את הוקטור בהצגה פולרית
ב. נתון וקטור שאורכו 10 מטרים ונמצא במישור בזווית 35 לציר, מצא/י את ההצגה הקרטזית של הוקטור במרחב x
ג. מצאי את הנגזרת של הוקטור $\vec{r} = (t^2, cos5t, 7)$

Solution

1. To answer this question, all we need to do is calculate the size of $\vec{r}$ and the angle. We also know that $r_x = 3$, $r_y = 4$. Using Eq.(2) to calculate the size of the given vector we get:

$$r = |\vec{r}\ | = \sqrt{r_x^2 + r_y^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5 \tag{6}$$

and the angle is:

$$\theta = \arctan\big(\frac{r_y}{r_x}\big) = \arctan\big(\frac{4}{3}\big) = 53.13^\circ \tag{7}$$

therefore, in polar coordinates, $\vec{r}$ can be written as:

$$\vec{r} = (5 \ , \ 53.13) \tag{8}$$

2. In this part of the question, we start with a vector in polar presentation, $\vec{r} = (10m \ , \ 35^{circ})$, and would like to present the given vector in cartesian presentation. We can do so by calculating the projection of the vector on the x and y axis.

$$r_x = r\cos(\theta) = 10\cos(35^\circ) = 8.19 \ m \quad , \quad r_y = r\sin(\theta) = 10\sin(35^\circ) = 5.73 \ m \tag{9}$$

so we get:

$$\vec{r} = (8.19 \ , \ 5.73) \ m \tag{10}$$

3. All we need to do is calculate the derivative of each term separately. By doing so, we obtain:

$$\dot{\vec{r}} = \frac{d}{dt}\Big(t^2 \ , \ \cos 5t \ , \ 7\Big) \quad \rightarrow \quad \vec{v} = \dot{\vec{r}} = (2t \ , \ -5\sin 5t \ , \ 0) \tag{11}$$

3 Question 2

מטוס טס מזרחה מעיר א' לעיר ב' המרוחקות זו מזו 488 ק"מ ב-45 דקות, לאחר מכן המטוס ממשיך דרומה מעיר ב' לעיר ג' המרוחקות זו מזו 966 ק"מ במשך 90 דקות. ניתן להתעלם מזמני העיכוב בנחיתה/המראה. עבור הטיסה כולה, מצאו את:

א. גודל ההעתק

ב. הכיוון בו ההעתק התבצע

ג. גודל וכיוון המהירות הממוצעת

Solution

1. The size of the displacement is the distance from the origin to the final position of the plane. If we know that the plane flew 488 km to the east and then 966 km to the south, we understand that the position of the plane at the end of the flight is $\vec{r} = 488\hat{x} - 966\hat{y}$. Therefore, the size of the displacement is:

$$|\vec{r}\,| = \sqrt{488^2 + 966^2} = 1082.266\ km \tag{12}$$

2. The direction of the displacement is calculated as usual:

$$\theta = \arctan(-\frac{966}{488}) = 296.8^\circ = -63.19^\circ \tag{13}$$

3. We know that the mean velocity is calculated using $\bar{v} = \frac{\Delta x}{\Delta t}$. In this question, the plane flew 45 minutes east and another 90 minutes south. Therefore, $\Delta t = 1.5 + 0.75 = 2.25\ h$, so we get:

$$\bar{v} = \frac{1082.266}{2.25} = 480.98\ \frac{km}{h} \tag{14}$$

the direction of the mean velocity is in the direction of the displacement, that is, $\theta = -63.19^\circ$

4 Question 3

מכים בכדור ביליארד במהירות בגודל $15\frac{m}{s}$ בכיוון $\theta = 30^\circ$

מה יהיה מיקום הכדור אחרי $t = 3s$?

כתבו את משוואות התנועה של הכדור בכל אחד מהצירים, מה יהיה המרחק של הכדור מכל אחד מהצירים בזמן $t = 3s$?

Solution

1. We can start by writing the velocity equation in the direction of the movement of the billiard:

$$r(t) = vt + x_0 \quad , \quad v = 15\ \frac{m}{s} \quad , \quad x_0 = 0 \quad \rightarrow \quad r(t) = 15t \tag{15}$$

so after $t = 3$ we position if the billiard is:

$$\vec{r} = (45m\ ,30°) \tag{16}$$

2. To find the position equation for the billiard projection on each of the axes, all we need to do is multiply $r(t)$ by $\cos 30$ or $\sin 30$ so we obtain the equations for x and y respectively. Therefore, we get the following:

$$r_x = 15t \cdot \cos 30 \quad , \quad r_y = 15t \cdot \sin 30 \quad \rightarrow \quad \vec{r}(t) = \begin{pmatrix} 15t \cdot \cos 30 \\ 15t \cdot \sin 30 \end{pmatrix} \tag{17}$$

so after $t = 3\ s$ we find that the distance of the ball in each of the axes is:

$$\vec{r}(t = 3\ s) = \begin{pmatrix} 45 \cdot \cos 30 \\ 45 \cdot \sin 30 \end{pmatrix} = \begin{pmatrix} 38.97\ m \\ 22.5\ m \end{pmatrix} \tag{18}$$

5 Question 4

1. כדור נזרק אופקית מבניין בגובה $H[m]$ במהירות אופקית $v[\frac{m}{s}]$ היכן ינחת הכדור?
2. כדור נזרק אופקית מבניין בגובה $H[m]$ נתון שהכדור נחת במרחק $x[m]$ מתחתית הבניין מה היא מהירות זריקת הכדור?
3. כדור נזרק אופקית מבניין בגובה $H[m]$ במהירות אופקית $v[\frac{m}{s}]$ נחת במרחק $x[m]$ מתחתית הבניין בטאו את גובה הניין על ידי x ו-v.

Solution

1. We start by defining the positive x axis to the right and the positive y axis to the top. Now we can break up the motion of the ball into two different axes. Along the y axis, the ball is in motion with constant acceleration g, that is, the ball is in free fall. In the x axis, the ball is in motion with constant velocity. The equations are given by:

$$y(t) = H + \frac{1}{2}gt^2 = H - 5t^2 \quad , \quad x(t) = vt \tag{19}$$

Now, to find the position of the ball at the time it landed, we need to know at what time it happens. We can find t_l since we know that $y(t_l) \equiv 0$ when the ball lands. Therefore:

$$y(t_l) \equiv 0 \quad \rightarrow \quad t_l = \sqrt{\frac{H}{5}} \tag{20}$$

by plugging the landing time, t_l, into $x(t)$ we find that the position of landing is:

$$x(t_l) = v\sqrt{\frac{H}{5}} \quad \rightarrow \quad \vec{r_l} = (v\sqrt{\frac{H}{5}}\ ,\ 0) \tag{21}$$

2. We can use the previous equation and isolate v, so we get:

$$v = x\sqrt{\frac{5}{H}} \tag{22}$$

3. All we need to do is isolate H from one of the previous equations. Therefore:

$$H = 5\Big(\frac{x}{v}\Big)^2 \tag{23}$$

6 Question 5

ממטוס הטס אופקית במהירות $v = 360\ km/h$ ובגובה $h = 980\ m$ מוטלת פצצה לעבר מטרה נייחת.

1.מה צריך להיות המרחק האופקי בניהם ברגע הטלת הפצצה על מנת להשיג פגיעה?

2.מה המרחק, גודלף וכיוון של המטוס מהמטרה ברגע שחרור הפצצה?

Solution

1. The first thing we can say is that the velocity of the bomb in the x axis is the same as the plane's velocity, that is, $v_x = 360\frac{km}{h} = 100\frac{m}{s}$. Therefore, the projection of the bomb's position is given by the following equation:

$$x(t) = 100t \tag{24}$$

The next step would be to evaluate how the distance that the bomb has passed along the x axis. To do so, we need to calculate the time it took the bomb to reach the ground, therefore:

$$y(t) = h - \frac{1}{2}gt^2 \quad \rightarrow \quad y(t_g) \equiv 0 \quad \rightarrow \quad t_g = \sqrt{\frac{980}{5}} = \sqrt{196} = 14\ s \tag{25}$$

plugging t_g into the position equation we get:

$$x(t = 14\ s) = 100 \cdot 14 = 1400\ m \tag{26}$$

This means that the pilot must release the bomb exactly 1400 meters before the target or he would miss the target.

2. Let's start by defining the origin at the location of the target. Therefore, the location of the plane is: $\vec{r_plane} = (-1400, 980)\ m$. Calculating the distance and direction between the plane and the target, we get the following:

$$d = |\vec{r}\,| = \sqrt{(-1400)^2 + 980^2} = 1708.91\ m \quad , \quad \theta = \arctan\big(-\frac{980}{1400}\big) = -35^\circ + 180^\circ = 145^\circ \tag{27}$$

where the 180 degrees were added since we know that the plane's position is in the second quadrature.

7 Question 6

כדור טניס נורה ממכונה בגובה הקרקע כלפי מעלה במהירות

$v\frac{m}{s}$

בזמן תעופתו, הכדור מואץ בגלל כוח סחף מהאוויר בצורה הבאה:

$\vec{a} = \alpha t\hat{x}$

כמו כן, קיימת כבידה בציר ה- y

1. מהו המיקום והמהירות ההתחלתי של הגוף?
2. מהי מהירות הכדור כפונקציה של הזמן בכל אחד מהצירים?
3. מהו מיקום הכדור כפונקציה של הזמן בכל אחד מהצירים?
4. מתי הכדור יגיע לשיא גובהו?
5. מתי יפגע הכדור בקרקע?

Solution

1. We can start by defining the origin of our coordinate system in such a way that the tennis ball starts its motion from the origin. The y axis points up, and the x axis is to the right. The initial position and initial velocity of the tennis ball is given by:

$$\vec{r}_0 = (0\ ,\ 0) \quad , \quad \vec{v}_0 = (0\ ,\ v)\frac{m}{s} \tag{28}$$

2. In the vertical axis (y), the tennis ball has constant acceleration of g. In the x axis, the motion is with changing velocity. The equations of velocity can be calculated by integrating the velocity of each axis, therefore, can be written in the following way:

$$v_y(t) = v(t) = v + gt = v - 10t \quad , \quad v_x(t) = v(t) = \int a_x dt = \frac{1}{2}\alpha t^2 \tag{29}$$

3. The equations for the position in each axis can be found by integrating the velocity that we have found previously, therefore:

$$r_y(t) = y(t) = vt + \frac{1}{2}gt^2 = vt - 5t^2 \quad , \quad r_x(t) = x(t) = \int v_x(t)dt = \frac{1}{6}\alpha t^3 \tag{30}$$

4. The tennis ball will reach the maximal height when the velocity in the vertical axis (y) is zero. Therefore:

$$v(t_{max})_y \equiv 0 \quad \rightarrow \quad v + gt_{max} = 0 \quad \rightarrow \quad t_{max} = -\frac{v}{g} = \frac{v}{10}\ s \tag{31}$$

5. To find the exact moment that the tennis ball reaches the ground, we need to find when $y(t) \equiv 0$, so we can write:

$$y(t) \equiv 0 \quad \rightarrow \quad t(v + \frac{1}{2}gt)t = 0 \quad \rightarrow \quad t_1 = t_{initial} = 0 \quad , \quad t_2 = -\frac{2v}{g} = \frac{v}{5}\ s \tag{32}$$