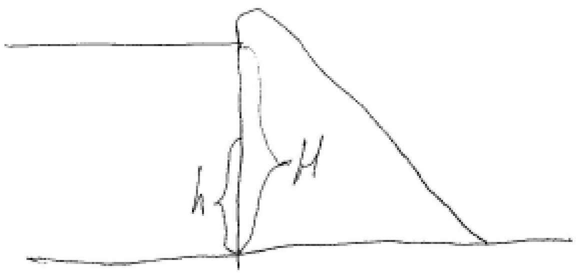
1. *Water is filled to a height H behind a dam of width w. The water pressure exerts a clockwise torque on the dam. If the torque were enough to cause the dam to break free from its foundation, the dam would pivot about its base. What is the magnitude of the torque about the base of the dam due to the water in the reservoir?*



$$p - p_{\text{atm}} = \rho g (H - h)$$
$$d\tau = h dF = h(p - p_{\text{atm}}) w dh$$
$$\tau = \int_0^H h \rho g (H - h) w dh = \tfrac{1}{6} \rho g w H^3$$

2. *On 8 May 1654, Otto von Guericke, mayor of Magdeburg and inventor of the air pump, demonstrated, in front of the Reichstag and the Emperor Ferdinand III, his celebrated Magdeburg hemispheres experiment. He evacuated air from a sphere, around 50 cm in diameter, made of two brass hemispheres. Thirty horses, in two teams of 15, could not separate the hemispheres until the valve was opened to equalize the air pressure. Determine the force required to pull the evacuated hemispheres apart. Hint: consider equilibrium of a hemispheric volume of air and show that the air pressure force on the hemispheric envelope is equal to the force on the base of the hemisphere.*

We have to calculate the total force the air excerts on a hemispheric envelope. Let us select a

hemispheric volume of air. The weight of air within the volume is negligible therefore the equilibrium condition implies that the force excerted by the surrounding air on the hemispheric envelope is equal to the force on the base, i.e.

$$F = \pi R^2 p_{\text{atm}} = \pi \cdot 0.25^2 \cdot 10^5 = 2 \cdot 10^4 \text{ N}$$

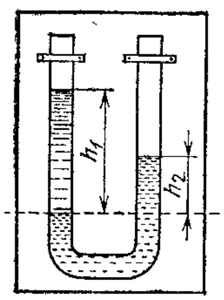
This force is required to pull the evacuated hemispheres apart.

3. *What fraction of the iceberg lies below the water level? The density of ice is 0.917 g/cm^3.*

   If V is the volume of the iceberg and the volume of the displaced water is V', then $\rho_{\text{water}} V' = \rho_{\text{ice}} V$. The fraction of ice beneath the water surface is

$$\frac{V'}{V} = \frac{\rho_{\text{ice}}}{\rho_{\text{water}}} = 0.917.$$

4. *A U-tube of uniform cross-sectional area and open to the atmosphere is partially filled with water. Kerosene is then poured into the left arm. If the equilibrium configuration of the tube is as shown in the figure, with h_2.=25 mm, determine the value of h_1 . Density of kerosene is 0.8 g/cm^3.*



The pressure at the same level should be equal:

$$\rho_{\text{kerosene}} h_1 = \rho_{\text{water}} h_2$$

$$h_1 = \frac{\rho_{\text{water}}}{\rho_{\text{kerosene}}} h_2 = 31 \text{ mm}$$

5. *King Hiero II of Syracuse suspected he was being cheated by the goldsmith to whom he had supplied the gold to make a* crown*. He asked Archimedes to find out if pure gold has been substituted by the same weight of electrum (alloy of gold and silver). Archimedes solved this problem by weighing the crown first in air and then in water. Suppose the weight of crown in air was 740 g and in water 690 g. What should Archimedes have told the king? Density of gold is 19.3 g/cm^3.*

The mass of water the crown displaced is $m = 740 - 690 = 50$ g. *Therefore the volume of the crown is 50 cm*^{3}*, which implies that the crown was made from substance with the density 740/50=14.8* g/cm^3. This is less than the density of gold therefore Archimedes should have told the king that he has been cheated.

6. *Prove Archimedes' principle by direct calculation of the pressure forces on the sides of a cube submerged in water (assume that the upper side of the cube is parallel to the surface of the water).*

$$p(h) = p_{\text{atm}} + \rho g h$$

$$F = a^2[p(h+a) - p(h)] = a^2 \cdot \rho g a = \rho g V$$