Projects Problems

April 2022

1 1006

An electronic transition in ions of ^{12}C leads to photon emission near $\lambda = 500\ nm$ ($h\nu = 2.5\ eV$). The ions are in thermal equilibrium at an ion temperature $kT = 20\ eV$, a density $n = 10^{24} m^{-3}$, and a non-uniform magnetic field which ranges up to $B = 1$ Tesla.

1. Briefly discuss broadening mechanisms which might cause the transition to have an observed width $\Delta\lambda$ greater thn that obtained for very small values of T, n and B.

2. For one of these mechanisms calculate the broadened width $\delta\lambda$ using order-of-magnitude estimates of needed parameters.

2 1008

Describe briefly each of the following effects or, in the case of rules, state the rule:

1. Auger effect

2. Anomalous Zeeman effect

3. Lamb shift

4. Land´e interval rule

5. Hund's rules for atomic levels

3 1024

An electron is confined to the interior of a hollow spherical cavity of radius R with impenetrable walls. Find an expression for the pressure exerted on the walls of the cavity by the electron in its ground state.

4 1036

We may generalize the semiclassical Bohr-Sommerfeld relation

$$\oint \vec{p} \cdot d\vec{r} = (n + \frac{1}{2})2\pi\hbar$$

(where the integral is along a closed orbit) to apply to the case where an electromagnetic field is present by replacing $\vec{p} \rightarrow \vec{p} - \frac{e\vec{A}}{c}$. Use this and the equation of motion for the linear momentum p to derive a quantized condition on the magnetic flux of a semiclassical electron which is in a magnetic field $\vec{B}$ in an arbitrary orbit. For electrons in a solid this condition can be restated in terms of the size S of the orbit in k-space. Obtain the quantization condition on S in terms of B. (Ignore spin effects)

5 1037

If a very small uniform-density sphere of charge is in an electrostatic potential $V(\vec{r})$, its potential energy is

$$U(\vec{r}) = V(\vec{r}) + \frac{r_0^2}{6}\nabla^2 V(\vec{r}) + ...$$

where $\vec{r}$ is the potential of the center of the charge and r_0 is its very small radius. The "Lamb shift" can be thourght of as the small correction to the energy of the hydrogen atom because the physical electron does have this property. If the r_0^2 term of U is treated as a very small perturbation compared to the Coulolmb interaction $V(\vec{r}) = -e^2/r$, what are the Lamb shifts for the $1s$ and $2p$ levels of the hydrogen atom? Express your result in terms of r_0 and fundamental constants. The unpretirbed wave functios are

$$\psi_{1s}(\vec{r}) = 2a_0^{-3/2} Exp(-r/a_0) Y_0^0$$

$$\psi_{2pm}(\vec{r}) = a_0^{-5/2} r \ Exp(-r/2a_0) Y_1^m / \sqrt{24}$$

where $a_0 = \hbar^2/m_e e^2$.

6 1045

The "plum pudding" model of the atom proposed by J. J. Thomson in the early days of atomic theory consisted of a sphere of radius a of positive charge of total value Ze. Z is an integer and e is the fundamental unit of charge. The electrons, of charge e, were considered to be point charges embedded in the positive charge.

1. Find the force acting on an electron as a function of its distance r from the center of the sphere for the element hydrogen.
2. What type of motion does the electron execute?
3. Find an expression for the frequency for this motion.

7 1046

Lyman alpha, the n = 1 to n = 2 transition in atomic hydrogen, is at $1215\mathring{A}$.

1. Define the wavelength region capable of photoionizing a H atom in the ground level (n = 1).
2. Define the wavelength region capable of photoionizing a H atom in the first excited level (n = 2).
3. Define the wavelength region capable of photoionizing a He^+ ion in the ground level (n = 1).
4. Define the wavelength region capable of photoionizing a He^+ ion in the first excited level (n = 2).

8 1049

Using the Bohar model of the atom,

1. derive an expression for the energy levels of the He^+ ion.
2. calculate the energies of the $l = 1$ state in a magnetic field, neglecting the electron spin.

9 1050

An atom has a nucleus of charge Z and one electron. The nucleus has a radius R, inside which the charge (protons) is uniformly distributed. We want to study the potential taking into account the finite size of the nucleus.

1. Calculate the potential taking into account the finite size of the nucleus.
2. Calculate the level shift due to the finite size of the nucleus for the $1s$ state of ^{208}Pb using perturbation theory, assuming that R is much smaller than the Bohr radius and approximating the wave function accordingly.

10 1054

What is meant by the fine structure and hyperfine structure of spectral lines? Discuss their physical origins. Give an example of each, including an estimate of the magnitude of the effect. Sketch the theory of one of the effects.

11 1058

Consider an atom formed by the binding of an Ω^- particle to a bare Pb nucleus (Z = 82). Calculate the energy splitting of the n=10, l = 9 level of this atom due to the spin-orbit interaction. The spin of the Ω particle is 3/2. Assume a magnetic moment of $\vec{\mu} = \frac{e\hbar}{2mc} g\vec{s}$ with $g = 2$ and $m = 1672\ Mev/c^2$. Note:

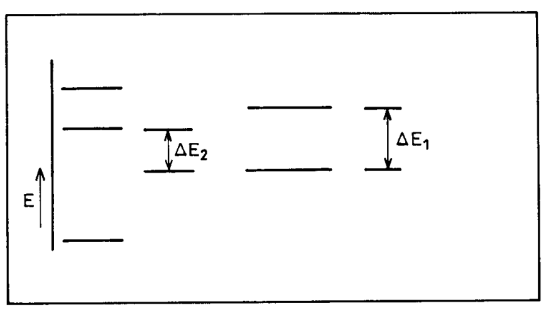
$$\langle \frac{1}{r^3} \rangle = \left(\frac{mc^2}{\hbar c} \right)^3 (\alpha Z)^3 \frac{1}{n^3 l(l+\frac{1}{2})(l+1)}$$

for a particle of mass m bound to a charge Z in hydrogen-like state of quantum numbers (n, l).

12 1069

The figure below shows the ground state and the set of n = 2 excited states of the helium atom. Reproduce the diagram in your answer giving

1. the spectroscopic notation for all 5 levels,
2. an explanation of the source of ∇E_1,
3. an explanation of the source of ∇E_2,
4. indicate the allowed optical transitions among these five levels.



13 1073

For helium atom the only states of spectroscopic interest are those for which at least one electron is in the ground state. It can be constructed from orthonormal orbits of the form $\Psi_{\pm}(1,2) = \frac{1}{\sqrt{2}}[\Phi_{1s}(1)\Phi_{nlm}(2) \pm \Phi_{nlm}(1)\Phi_{1s}(s)]\times$ spin wave function. The para-states correspond to the + sign and the ortho-states to the sign.

1. Determine for which state the ortho- or the corresponding para-state has the lowest energy. (i.e. most negative).

2. Present an argument showing for large n that the energy difference between corresponding ortho- and para-states should become small.

14 1100

Consider a helium atom with a 1s3d electronic configuration. Sketch a series of energy-level diagrams to be expected when one takes successively into account:

1. only the Coulomb attraction between each electron and the nucleus,
2. the electrostatic repulsion between the electrons,
3. spin-orbit coupling,
4. the effect of a weak external magnetic field.

15 1109

Sketch a derivation of the "Land´e g-factor", i.e. the factor determining the effective magnetic moment of an atom in weak fields.

16 1115

Give a brief description of the Stern-Gerlach experiment and answer the following questions:

1. Why must the magnetic field be inhomogeneous?
2. How is the inhomogeneous field obtained?
3. What kind of pattern would be obtained with a beam of hydrogen atoms in their ground state? Why?
4. What kind of pattern would be obtained with a beam of mercury atoms (ground state $^1S_0$)? Why?

17 1116

The atomic number of aluminum is 13.

1. What is the electronic configuration of Al in its ground state?
2. What is the term classification of the ground state? Use standard spectroscopic notation and explain all superscripts and subscripts.
3. Show by means of an energy-level diagram what happens to the ground state when a very strong magnetic field (Paschen-Back region) is applied. Label all states with the appropriate quantum numbers and indicate the relative spacing of the energy levels.

18 1118

A particular spectral line corresponding to a $J = 1 \rightarrow J = 0$ transition is split in a magnetic field of 1000 gauss into three components separated by $0.0016\mathring{A}$. The zero field line occurs at $1849\mathring{A}$.

1. Determine whether the total spin is in the J = 1 state by studying the g-factor in the state.
2. What is the magnetic moment in the excited state?

19 1123

1. Assuming that the two protons of the H_2^+ molecule are fixed at their normal separation of $1.06\mathring{A}$, sketch the potential energy of the electron along the axis passing through the protons.
2. Sketch the electron wave functions for the two lowest states in H_2^+, indicating roughly how they are related to hydrogenic wave functions. Which wave function corresponds to the ground state of H_2^+, and why?

20 1124

Given the radial part of the Schrodinger equation for a central force field V (r):

$$-\frac{\hbar^2}{2\mu}\frac{1}{r^2}\frac{d}{dr}\Big(r^2\frac{d\Psi(r)}{dr}\Big) + \Big[V(r) + \frac{l(l+1)\hbar^2}{2^2}\Big]\Psi(r) = E\Psi(r)$$

consider a diatomic molecule with nuclei of masses m_1 and m_2. A good approximation to the molecular potential is given by

$$V(r) = -2V_0\Big(\frac{1}{\rho} - \frac{1}{2\rho^2}\Big)$$

where $\rho = r/a$, a with a being some characteristic length parameter.

1. By expanding around the minimum of the effective potential in the Schrodinger equation, show that for small B the wave equation reduces to that of a simple harmonic oscillator with frequency

   $$\omega = \Big[\frac{2V_0}{\mu a^2(1+B)^3}\Big]^{1/2}$$

   where

   $$B = \frac{l(l+1)\hbar^2}{2a^2V_0}$$

2. Assuming $\hbar^2/2\mu a V_0$, find the rotational, vibrational and rotation-vibrational energy levels for small oscillations.

21 1132

1. Using hydrogen atom ground state wave functions (including the electron spin) write wave functions for the hydrogen molecule which satisfy the Pauli exclusion principle. Omit terms which place both electrons on the same nucleus. Classify the wave functions in terms of their total spin.

2. Assuming that the only potential energy terms in the Hamiltonian arise from Coulomb forces discuss qualitatively the energies of the above states at the normal internuclear separation in the molecule and in the limit of very large internuclear separation.

3. What is meant by an "exchange force"?

22 1134

A ${}^{7}N_{14}$ nucleus has nuclear spin I = 1. Assume that the diatomic molecule N2 can rotate but does not vibrate at ordinary temperatures and ignore electronic motion. Find the relative abundance of ortho and para molecules in a sample of nitrogen gas. (Ortho = symmetric spin state; para = antisymmetric spin state).

23 1135

In HCl a number of absorption lines with wave numbers (in $1/cm$) 83.03, 103.73, 124.30, 145.03, 165.51, and 185.86 have been observed. Are these vibrational or rotational transitions? If the former, what is the characteristic frequency? If the latter, what J values do they correspond to, and what is the moment of inertia of HCl? In that case, estimate the separation between the nuclei.