Homework 5

- Throughout this homework set you are asked to solve the questions in Coulomb gauge.

Question 1

Find the magnetic field for a sphere with permeability constant μ in a homogeneous external magnetic field $B_0\hat{z}$.

Question 2

The surface charge density of a spherical shell of radius R is given by

$$\sigma_0(\theta) = \sigma_0 \cos\theta.$$

The sphere is rotating with constant angular velocity ω. Find the vector potential everywhere.

Hint: Use the result from the previous homework, where you found that

$$\frac{1}{|\vec{r}-\vec{r}'|} = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \frac{1}{2\ell+1} \frac{r_<^\ell}{r_>^{\ell+1}} Y_{\ell m}^*(\theta',\varphi')\, Y_{\ell m}(\theta,\varphi), \tag{1}$$

to compute $\vec{A}$ directly.

Question 3 - The classical electron model

Assume that the electron is a hollow conducting shell of radius R, charge uniformly with a total charge e, and rotating at some constant angular velocity ω.

1. Find the electric potential and field everywhere in space.
2. Show that the current density on the shell is

$$\vec{K}(\vec{r}) = \sigma\omega R \sin\theta \hat{\boldsymbol{\varphi}}.$$

3. Compute the vector potential. Show that the system has a magnetic dipole moment $\vec{m} = eR^2\vec{\omega}/(3c)$.
   Hint: Once again, use equation (1) to directly compute the potential.
4. Find the magnetic field $\vec{B}(\vec{r})$, and show that the magnetic field inside the shell is constant.
5. Compute the total energy stored in the EM field.
6. Compute the total angular momentum stored in the EM field.
7. Assume that the Electron's mass is related to its stored energy through $m = \mathcal{E}/c^2$ and that its angular momentum is $\hbar/2$. Under these assumption, evaluate R and the velocity $v = \omega R$. Do the values of R, v make sense?